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by

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Tight upper bound for the maximal quantum value of the Svetlichny operators

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Abstract

It is a challenging task to detect genuine multipartite nonlocality (GMNL). In this paper, the problem is considered via computing the maximal quantum value of Svetlichny operators for three-qubit systems and a tight upper bound is obtained. The constraints on the quantum states for the tightness of the bound are also presented. The approach enables us to give the necessary and sufficient conditions of violating the Svetlichny inequalities (SI) for several quantum states, including the white and color noised GHZ states. The relation between the genuine multipartite entanglement concurrence and the maximal quantum value of the Svetlichny operators for mixed GHZ class states is also discussed. As the SI is useful for the investigation on GMNL, our results give an effective and operational method to detect the GMNL for three-qubit mixed states.

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I. INTRODUCTION

Quantum nonlocality, which is incompatible with the local hidden variable (LHV) theory, can be revealed via violations of various Bell inequalities [1–8]. It has been recognized that quantum nonlocality is not only a puzzling aspect of nature, but also an important resource for quantum information processing, such as building quantum protocols to decrease communication complexity [9, 10] and providing secure quantum communication [11, 12].

For the multipartite case, quantum nonlocality displays much richer and more complex structures than the bipartite case [13]. One can distinguish qualitatively different kinds of nonlocality. In this manuscript, we consider the tripartite case. If Alice, Bob and Charlie perform measurement X , Y and Z on the three subsystems, respectively, with outcomes a , b and c , and the probability correlations $P(abc|XYZ)$ among the measurement outcomes can be written as

$$P(abc|XYZ) = \sum_{\lambda} q_{\lambda} P_{\lambda}(a|X) P_{\lambda}(b|Y) P_{\lambda}(c|Z), \quad (1)$$

where $0 \leq q_{\lambda} \leq 1$ and $\sum_{\lambda} q_{\lambda} = 1$, then the state measured is called three local. Otherwise the state is non-three local. Svetlichny [14] pointed out that some correlations can be written in the hybrid local-nonlocal (or bi-LHV) form,

$$\begin{aligned} & P(abc|XYZ) \\ &= \sum_{\lambda} q_{\lambda} P_{\lambda}(ab|XY) P_{\lambda}(c|Z) + \sum_{\mu} q_{\mu} P_{\mu}(ac|XZ) P_{\mu}(b|Y) + \sum_{\nu} q_{\nu} P_{\nu}(bc|YZ) P_{\nu}(a|X), \quad (2) \end{aligned}$$

where $0 \leq q_{\lambda}, q_{\mu}, q_{\nu} \leq 1$ and $\sum_{\lambda} q_{\lambda} + \sum_{\mu} q_{\mu} + \sum_{\nu} q_{\nu} = 1$. Such correlations are called genuine tripartite nonlocal. Svetlichny has formulated a hybrid nonlocal-local realism based inequality [14]: a stronger kind of inequality for a three-qubit system where two of the qubits are assumed to be non-locally correlated, but they are locally correlated to the third, with an ensemble average over all such possible combinations. The violation of the Svetlichny inequality (SI) is a signature of genuine tripartite nonlocality. We refer to [15–19] for more details.

The question studied in this paper is that for an arbitrary (pure or mixed) three-qubit state, how one checks whether the SI is violated or not. The authors in [20] has considered this problem for pure GHZ class states and W class states, and explicit formula for the maximal values of the Svetlichny operator over all the measurement observables are obtained.

In [21] the authors have provided analytical and numerical prescriptions for detecting the maximum violation of the SI for pure and mixed Gaussian states of continuous variable systems.

In this brief report we consider the computation of the maximal quantum value of the Svetlichny operators for any three-qubit systems. We present a tight upper bound for the maximal quantum value. We also provide the constraints on the quantum state for the tightness of the bound. Moreover, the sufficient and necessary condition of violating the SI for several quantum states is given, including the white and color noised GHZ states. We also discuss the relation between the genuine tripartite entanglement concurrence and the maximal quantum value of the Svetlichny operators for mixed GHZ class states. As the SI is powerful to investigate GMNL, our results give an effective and operational way to detect the GMNL for three-qubit mixed states.

II. THE SVETLICHNY INEQUALITY AND THE MAXIMAL VIOLATION

We start with a short review of the Svetlichny inequality [14]. The Svetlichny operators \mathcal{S} are defined as follows:

$$\mathcal{S} = A \otimes [(B + B') \otimes C + (B - B') \otimes C'] + A' \otimes [(B - B') \otimes C - (B + B') \otimes C'], \quad (3)$$

where A, A', B, B', C and C' are observables of the form $G = \vec{g} \cdot \vec{\sigma} = \sum_k g_k \sigma_k$, $G \in \{A, A', B, B', C, C'\}$ and $\vec{g} \in \{\vec{a}, \vec{a}', \vec{b}, \vec{b}', \vec{c}, \vec{c}'\}$ correspondingly, σ_k ($k = 1, 2, 3$) are the Pauli matrices, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, $\vec{g} = (g_1, g_2, g_3)$ is a 3-dimensional real unit vector. For any 3-qubit state $|\Psi\rangle$ with the bi-LHV form (2), the mean value of the Svetlichny operators is bounded as follows [14]:

$$|\langle \Psi | \mathcal{S} | \Psi \rangle| \leq 4. \quad (4)$$

Theorem: For any three-qubit quantum state ρ , the maximum quantum value $Q(\mathcal{S})$ of the Svetlichny operator \mathcal{S} defined in (3) satisfies

$$Q(\mathcal{S}) \equiv \max |\langle \mathcal{S} \rangle_\rho| \leq 4\lambda_1,$$

where $\langle \mathcal{S} \rangle_\rho = \text{Tr}(\mathcal{S}\rho)$, λ_1 is the maximum singular value of the matrix $M = (M_{j,ik})$, with $M_{ijk} = \text{tr}[\rho(\sigma_i \otimes \sigma_j \otimes \sigma_k)]$, $i, j, k = 1, 2, 3$.

To prepare for the argument we first give the following result.

Lemma: Let A be a rectangular matrix of size $m \times n$. For any vectors $\vec{x} \in \mathbb{R}^m$ and $\vec{y} \in \mathbb{R}^n$ we have that

$$|\vec{x}^T A \vec{y}| \leq \lambda_{max} |\vec{x}| |\vec{y}|, \quad (5)$$

where λ_{max} is the largest singular value of the matrix A . The equality holds when \vec{x} and \vec{y} are the corresponding singular vectors of A with respect to λ_{max} .

Proof. By the singular value decomposition, there exist two unitary matrices U and V such that $A = U^T \Sigma V$, where Σ has only nonzero entries along the diagonal. Therefore, we may assume that $A = \Sigma$ and consider only the following form, $G(\vec{x}, \vec{y}) = \sum_i a_i x_i y_i$, where $a_1 \geq a_2 \geq \dots \geq a_n > 0$. Using the Cauchy-Schwarz inequality for the inner product $\langle \vec{x}, \vec{y} \rangle := G(\vec{x}, \vec{y})$, we have that

$$\begin{aligned} |G(\vec{x}, \vec{y})| &\leq G(\vec{x}, \vec{x})^{1/2} G(\vec{y}, \vec{y})^{1/2} \\ &= \left(\sum_i a_i x_i^2 \right)^{1/2} \left(\sum_i a_i y_i^2 \right)^{1/2} \\ &\leq a_1 \left(\sum_i x_i^2 \right)^{1/2} \left(\sum_i y_i^2 \right)^{1/2}, \end{aligned}$$

where a_1 corresponds to λ_{max} in (5). ■

Proof of the theorem. By definition we have that

$$\begin{aligned} \mathcal{S} &= A \otimes [(B + B') \otimes C + (B - B') \otimes C'] + A' \otimes [(B - B') \otimes C - (B + B') \otimes C'] \\ &= \sum_{i,j,k} [a_i(b_j + b'_j)c_k + a_i(b_j - b'_j)c'_k + a'_i(b_j - b'_j)c_k - a'_i(b_j + b'_j)c'_k] \sigma_i \otimes \sigma_j \otimes \sigma_k. \end{aligned}$$

Then

$$\begin{aligned} \langle \mathcal{S} \rangle_\rho &= \sum_{i,j,k} [a_i(b_k + b'_k)c_j + a_i(b_k - b'_k)c'_j + a'_i(b_k - b'_k)c_j - a'_i(b_k + b'_k)c'_j] \text{tr} [\rho(\sigma_i \otimes \sigma_k \otimes \sigma_j)] \\ &= \sum_{i,j,k} [a_i(b_k + b'_k)c_j + a_i(b_k - b'_k)c'_j + a'_i(b_k - b'_k)c_j - a'_i(b_k + b'_k)c'_j] M_{ij,k} \\ &= (\vec{a} \otimes \vec{c} - \vec{a}' \otimes \vec{c}')^T M(\vec{b} + \vec{b}') + (\vec{a} \otimes \vec{c}' + \vec{a}' \otimes \vec{c})^T M(\vec{b} - \vec{b}'). \end{aligned}$$

Now denote by θ_a (resp. θ_b, θ_c) the angle between \vec{a} and \vec{a}' (resp. \vec{b} and \vec{b}' , \vec{c} and \vec{c}'), we

then have

$$\begin{aligned}
|\vec{b} + \vec{b}'|^2 &= 2 + \langle \vec{b}, \vec{b}' \rangle = 2 + 2 \cos \theta_b = 4 \cos^2 \frac{\theta_b}{2}, \\
|\vec{b} - \vec{b}'|^2 &= 2 + \langle \vec{b}, \vec{b}' \rangle = 2 - 2 \cos \theta_b = 4 \sin^2 \frac{\theta_b}{2}, \\
|\vec{a} \otimes \vec{c} - \vec{a}' \otimes \vec{c}'|^2 &= 2 - 2 \langle \vec{a}, \vec{a}' \rangle \langle \vec{c}, \vec{c}' \rangle = 2 - 2 \cos \theta_a \cos \theta_c, \\
|\vec{a} \otimes \vec{c}' + \vec{a}' \otimes \vec{c}|^2 &= 2 + 2 \langle \vec{a}, \vec{a}' \rangle \langle \vec{c}, \vec{c}' \rangle = 2 + 2 \cos \theta_a \cos \theta_c.
\end{aligned}$$

Let θ_{ac} be the principal angle such that $\cos \theta_a \cos \theta_c = \cos \theta_{ac}$. Therefore

$$\begin{aligned}
|\vec{a} \otimes \vec{c} - \vec{a}' \otimes \vec{c}'|^2 &= 4 \sin^2 \frac{\theta_{ac}}{2}, \\
|\vec{a} \otimes \vec{c}' + \vec{a}' \otimes \vec{c}|^2 &= 4 \cos^2 \frac{\theta_{ac}}{2}.
\end{aligned}$$

It follows from the Lemma (5) and trigonometric identities that

$$\begin{aligned}
|\langle \mathcal{S} \rangle_\rho| &\leq \lambda_1 \left(|\vec{a} \otimes \vec{c} - \vec{a}' \otimes \vec{c}'| |\vec{b} + \vec{b}'| + |\vec{a} \otimes \vec{c}' + \vec{a}' \otimes \vec{c}| |\vec{b} - \vec{b}'| \right) \\
&= 4\lambda_1 \left(\left| \sin \frac{\theta_{ac}}{2} \cos \frac{\theta_b}{2} \right| + \left| \cos \frac{\theta_{ac}}{2} \sin \frac{\theta_b}{2} \right| \right) \\
&= 4\lambda_1 \left| \sin \left(\frac{\theta_{ac} \pm \theta_b}{2} \right) \right| \\
&\leq 4\lambda_1,
\end{aligned} \tag{6}$$

which proves the inequality. \blacksquare

Let's look at when the equality holds. Actually, to saturate the upper bound in the Theorem, one can always select $\theta_{ac} \pm \theta_b = \pi$ or $-\pi$ by setting proper measurement directions of B and B' such that the last inequality in (6) becomes an equality. Then from the Lemma, we have that the first inequality in (6) saturates if the degeneracy of λ_1 is more than one, and corresponding to λ_1 there are two 9-dimensional singular vectors taking the form of $\vec{a} \otimes \vec{c} - \vec{a}' \otimes \vec{c}'$ and $\vec{a} \otimes \vec{c}' + \vec{a}' \otimes \vec{c}$, respectively.

The next examples show that the upper bound is tight.

Example 1: We consider the mixture of the white noise and the three-qubit GHZ-class states, which is given by

$$\rho = p |\psi_{gs}\rangle \langle \psi_{gs}| + \frac{1-p}{8} I, \tag{7}$$

where I is identity matrix, $|\psi_{gs}\rangle = \cos \theta |000\rangle + \sin \theta |11\rangle (\cos \theta_3 |0\rangle + \sin \theta_3 |1\rangle)$, and $0 \leq p \leq 1$.

The matrix M is of the form,

$$M = p \begin{pmatrix} a & 0 & b & 0 & -a & 0 & 0 & 0 & 0 \\ 0 & -a & 0 & -a & 0 & -b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & d \end{pmatrix}, \quad (8)$$

where $a = 2 \cos \theta \sin \theta \sin \theta_3$, $b = 2 \cos \theta \cos \theta_3 \sin \theta$, $c = 2 \cos \theta_3 \sin^2 \theta \sin \theta_3$ and $d = \cos^2 \theta + \sin^2 \theta \cos^2 2\theta_3$.

The singular values of the matrix M are $p \cdot |\sin 2\theta| \sqrt{1 + \sin^2 \theta_3}$, $p \cdot |\sin 2\theta| \sqrt{1 + \sin^2 \theta_3}$, and $p \cdot \sqrt{1 - \sin^2 2\theta \sin^2 \theta_3}$. Hence the upper bound of the maximal value of the Svetlichny operators is

$$Q(\mathcal{S}) \equiv \max |\langle \mathcal{S} \rangle_\rho| \leq 4\lambda_1 = 4p \cdot \max_{\theta, \theta_3} \{ |\sin 2\theta| \sqrt{1 + \sin^2 \theta_3}, \sqrt{1 - \sin^2 2\theta \sin^2 \theta_3} \}. \quad (9)$$

One finds that $p \cdot \sqrt{1 - \sin^2 2\theta \sin^2 \theta_3}$ is always less than one. Thus to violate the SI, we can just take $\lambda_1 = p \cdot |\sin 2\theta| \sqrt{1 + \sin^2 \theta_3}$. To prove that the upper bound is saturated for the mixed state ρ in (7), we set, for convenience,

$$x = -\frac{\sin 2\theta \sin \theta_3}{\sin^2 2\theta (1 + \sin^2 \theta_3)}, \quad y = -\frac{\sin 2\theta \cos \theta_3}{\sin^2 2\theta (1 + \sin^2 \theta_3)}.$$

The singular vectors corresponding to $\lambda_{1,2}$ can be selected as $(0, x, 0, x, 0, y, 0, 0, 0)^T$ and $(-x, 0, -y, 0, x, 0, 0, 0, 0)^T$, which can be directly decomposed as

$$(1, 0, 0)^T \otimes (0, x, 0)^T + (0, -1, 0)^T \otimes (-x, 0, -y)^T$$

and

$$(1, 0, 0)^T \otimes (-x, 0, y)^T - (0, -1, 0)^T \otimes (0, x, 0)^T.$$

Then we can set

$$\vec{a} = (1, 0, 0)^T, \quad \vec{a}' = (0, -1, 0)^T, \quad \vec{c} = \frac{1}{\sqrt{x^2 + y^2}} (-x, 0, -y)^T, \quad \vec{c}' = (0, 1, 0)^T,$$

and set \vec{b} and \vec{b}' to be unit vectors s.t. $\theta_{ac} + \theta_b = 2\pi$. With the above settings one can find that each of the inequalities in the proof of the theorem becomes equal, which means that the upper bound is saturated for ρ .

Note that if we set $p = 1$, then the maximal value is in accordance with the main result in [20]. However, the theorem in this manuscript fits for arbitrary mixed three-qubit

systems. By the analysis above, one obtains that ρ in (7) will violate the SI if and only if $1/|\sin 2\theta|\sqrt{1 + \sin^2 \theta_3} < p \leq 1$. If we set $\theta_3 = \frac{\pi}{2}$ and $\theta = \frac{\pi}{4}$, the state ρ in (7) is exactly the ρ_{GHZ} : the mixture of the GHZ state $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and white noise. From our Theorem we have that ρ_{GHZ} will violate the SI if and only if $0.707107 < p \leq 1$, which is in accordance with the result in [13] from numerical optimization. By [22] we have that ρ_{GHZ} will be genuinely maximally entangled (GME) if and only if $0.428571 < p \leq 1$, see Fig. 1.

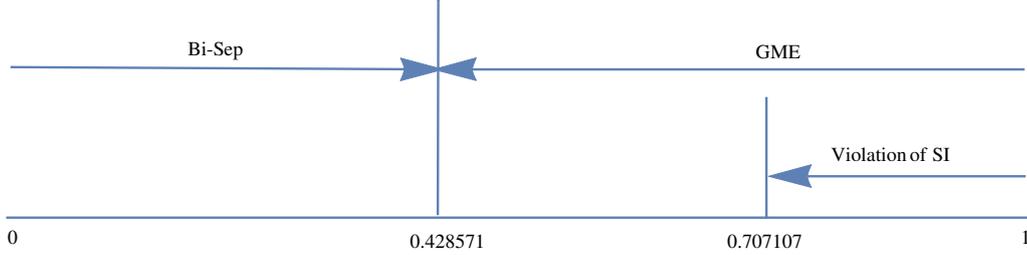


FIG. 1: Consider the mixture of GHZ state and white noise: $\rho_{GHZ} = p|GHZ\rangle\langle GHZ| + \frac{1-p}{8}I$, where $|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. By [22] we have that ρ_{GHZ} will be GME if and only if $0.428571 < p \leq 1$, while by our theorem one obtains that ρ_{GHZ} will violate the SI if and only if $0.707107 < p \leq 1$.

Example 2: Consider the quantum state $\sigma_A(\rho)$ presented in [23]. Set $N = 3$ and $d = 2$. Then $\sigma_A(\rho)$ is a three-qubit state,

$$\sigma_A(\rho) = p|GHZ\rangle\langle GHZ| + \frac{1-p}{4}I_2 \otimes \tilde{I}, \quad (10)$$

where I_2 stands for the 2×2 identity matrix and $\tilde{I} = \text{diag}(1, 0, 0, 1)$. As analyzed in [23], $\sigma_A(\rho)$ admits bi-local hidden models for $0 \leq p \leq 0.416667$, and is a GME states for $\frac{1}{3} < p \leq 1$. By our theorem, one gets

$$M = \begin{pmatrix} p & 0 & 0 & 0 & -p & 0 & 0 & 0 & 0 \\ 0 & -p & 0 & -p & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

and can further check that the upper bound is $4\sqrt{2}p$ and is saturated, which means that $\sigma_A(\rho)$ will be GMNL for $0.707107 < p \leq 1$, as shown in Fig. 2.

In the following we consider the relation between the GME concurrence and the SI bound obtained above. The GME concurrence is proved to be a well defined measure [25, 26]. For

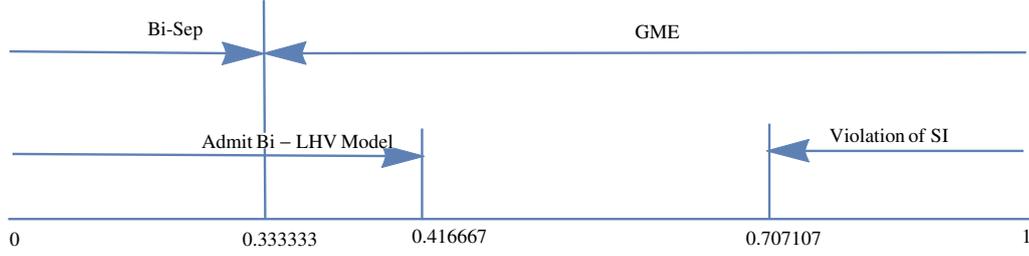


FIG. 2: Consider the mixture of GHZ state and color noise: $\sigma_A(\rho) = p|GHZ\rangle\langle GHZ| + \frac{1-p}{4}I_2 \otimes \tilde{I}$. By [23], one has that $\sigma_A(\rho)$ admit bi-local hidden models for $0 \leq p \leq 0.416667$, and are GME for $\frac{1}{3} < p \leq 1$, while by our theorem one obtains that $\sigma_A(\rho)$ will be GMNL for $0.707107 < p \leq 1$.

a tripartite pure state $|\psi\rangle$, the GME concurrence is defined by

$$C_{GME}(|\psi\rangle) = \sqrt{\min\{1 - \text{tr}(\rho_1^2), 1 - \text{tr}(\rho_2^2), 1 - \text{tr}(\rho_3^2)\}},$$

where ρ_i is the reduced matrix for the i th subsystem. For mixed states ρ , the GME concurrence is then defined by the convex roof,

$$C_{GME}(\rho) = \min_{\{p_\alpha, |\psi_\alpha\rangle\}} \sum p_\alpha C_{GME}(|\psi_\alpha\rangle). \quad (12)$$

The minimum is taken over all pure ensemble decompositions of ρ .

In [24] we have already found a lower bound for GME concurrence for three-qubit quantum systems. For a three-qubit state ρ , the GME concurrence satisfies the following inequality [24],

$$C_{GME}(\rho) \geq \sqrt{\frac{1}{8}\|M\|_{HS}^2 - \frac{1}{2}}, \quad (13)$$

where $\|M\|_{HS}$ stands for the Hilber-Schmidt norm or 2-norm of matrix M . For the state (7), we have

$$\begin{aligned} C_{GME}(\rho) &\geq \sqrt{\frac{1}{8}\|M\|_{HS}^2 - \frac{1}{2}} = \sqrt{\frac{1}{8}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) - \frac{1}{2}} \\ &= \sqrt{\frac{1}{64}Q^2(\mathcal{S}) + \frac{1}{8}(p^2 - p^2 \sin^2 2\theta \sin^2 \theta_3) - \frac{1}{2}}, \end{aligned} \quad (14)$$

which presents an explicit relation satisfied by the GME concurrence and the maximal value of the SI operators.

III. CONCLUSIONS AND DISCUSSIONS

We have presented a quantitative analysis of the genuine tripartite nonlocality for any three-qubit quantum systems via effective computation of the maximal quantum value of the Svetlichny operators. Our method provides a tight upper bound for the maximal quantum value. The tightness of the upper bound is investigated through several noisy quantum states. Our result works not only for pure states but also for mixed states. We have also discussed the relation between the GME concurrence and the maximal quantum value of the Svetlichny operators for mixed GHZ class states. Since the SI is powerful in detecting GMNL, our results give an effective and operational way to investigate the GMNL for three-qubit mixed states. The method presented in this manuscript can also be used to compute the maximal quantum value of the Svetlichny operators for the continuous quantum systems.

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