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Discriminating bipartite mixed states by local operations

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Unambiguous state discrimination of two mixed bipartite states via local operations and classical communications (LOCC) is studied and compared with the result of a scheme realized via global measurement. We show that the success probability of a global scheme for mixed-state discrimination can be achieved perfectly by the local scheme. In addition, we simulate this discrimination via a pair of pure entangled bipartite states. This simulation is perfect for local rather than global schemes due to the existence of entanglement and global coherence in the pure states. We also prove that LOCC protocol and the sequential state discrimination (SSD) can be interpreted in a unified view. We then hybridize the LOCC protocol with three protocols (SSD, reproducing and broadcasting) relying on classical communications. Such hybridizations extend the gaps between the optimal success probability of global and local schemes, which can be eliminated only for the SSD rather than the other two protocols.

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I. INTRODUCTION

Since useful quantum information is encoded in quantum states, state discrimination is one of the most crucial research topics in quantum information processing [1]. In particular, the unambiguous discrimination among linearly independent nonorthogonal quantum states is of fundamental significance in quantum information theory [2–7]. For the simplest state discrimination, one prepares a qubit in one of two known nonorthogonal states, $|\Psi_1\rangle$ and $|\Psi_2\rangle$, and sends it to an observer Alice. Alice's task is to determine the state she received by positive operator-valued measure (POVM). The measurement gives rise to three possible outcomes, $|\Psi_1\rangle$, $|\Psi_2\rangle$, and *inconclusive*, in which the last one is the price for perfect discrimination. Such unambiguous state discriminations play an important role in quantum key distribution [6] and the study of quantum correlations [8–10].

For multipartite quantum states, the measurement strategies can be classified into two types: global and local. The authors in Refs. [11–13] investigated nonorthogonal bipartite pure states discriminated via local operations. The observer Alice applies the discrimination operation on the first particle first. If she succeeds, the procedure ends. Otherwise, she sends the state to the next observer, Bob. Bob takes his discrimination operation on another particle. It is found that there exist protocols whose optimal state discrimination of local operations and classical communications (LOCC) are as good as global schemes.

To find out the essential role played by local schemes in state discrimination, we construct a pair of mixed bipartite states comprising two orthogonal vectors mixed with each other via classical probabilities, which contain no entanglement or global coherence. These mixed-state discrimination problems have given rise to many novel outcomes by global scheme in Ref. [14]. It is found that the optimal successful probability of global mixed state discrimination can also be achieved perfectly by the local scheme, which is observed in pure state cases [11].

In order to see the essential difference in identifying pure and mixed states, we can simulate the above-mentioned (separable) mixed-state discrimination by an entangled and globally coherent state. We find that this simulation is bound to be perfect for local scheme, since local POVMs eliminate the entanglement and global coherence that are critical recourses encoded in the pure states. Thus, the pure-state scheme does not necessarily show superiority to mixed ones. For the global scheme, successful simulation only occurs for a few special cases. Generally, the mixed-state protocol is inferior to the pure-entangled-state one.

Another scheme is the sequential unambiguous state discrimination (SSD) originated from one of the theories to extract information from a quantum system by multiple observers [15–17] and put forward by Bergou *et al.* in Ref. [15]. It is shown in Refs. [15, 18] that SSD is useful in quantum communication schemes (e.g., the B92 quantum cryptography protocol [5]). The optimal success probability of SSD was provided analytically in Ref. [15] and demonstrated experimentally [19]. Further investigations on the optimized success probability of SSD with global measurements are reported for both the pure states [18, 20, 21] and mixed states [14].

An interesting topic is to study the relationship between different tasks in quantum information. In this paper, we prove that SSD and LOCC can be interpreted in a unified view, despite the essential distinction between the two protocols: the classical communication is

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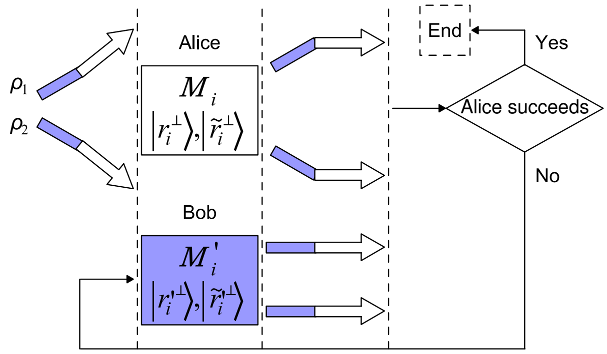

FIG. 1: Protocol for local mixed-state discrimination. First, a bipartite mixed quantum state ρ_i $(i = 1, 2)$ prepared with the prior probability P_i is sent to Alice. Alice performs unambiguous discrimination on the state via optimal local POVMs $\{M_i\}$ $(i = 0,\ 1,\ 2)$ on the subspace spanned by the basis $\{|r_i\rangle, |\tilde{r}_i\rangle\}$. Then, if Alice succeeds in discriminating the states optimally, the procedure ends; otherwise the state is sent to the other observer, Bob, who will discriminate the state optimally by the POVMs on the other subspace spanned by $\{|r_i'\rangle, |\tilde{r}_i'\rangle\}$.

forbidden in the former one but required in the latter.

Different from SSD, another two protocols called reproducing and discrimination after broadcasting, which allow classical communications, were discussed in Refs. [14, 15, 18] and compared with the result of SSD. It has been found that SSD performs better than the other two. It would be interesting to consider the compatibility of LOCC and these three protocols (SSD, reproducing and discrimination after broadcasting) for our bipartite systems. In order to see the effects of different information tasks on the gap between local and global schemes, we hybridize the LOCC protocol with these three protocols. We show that the optimal successful probability of global SSD can be attained by local SSD for some special cases. In contrast, the local scheme is inferior to the global one for the other two protocols.

The paper is organized as follows. In Sec. II, we present the result of locally discriminating mixed bipartite states, which is found to be equivalent to one of the global schemes. In Sec. III, we simulate mixed-state discrimination by entangled pure states via local and global schemes, respectively. In Sec. IV, we present a unified view of SSD and LOCC protocols. In Sec. V, we study the hybridization of LOCC and the other three protocols. We summarize in the last section.

II. MIXED-STATE DISCRIMINATION VIA LOCAL OPERATIONS

We first consider the procedure of mixed-state discrimination via local operations, (see Fig. 1). One prepares an ensemble of two mixed bipartite separable states ρ_i with *a priori* probability P_i, $(i = 1, 2,\ P_1 + P_2 = 1)$. The state ρ_i has a spectral decomposition,

$$\rho_i = r_i|r_i\rangle\langle r_i|\otimes|r_i'\rangle\langle r_i'| + \tilde{r}_i|\tilde{r}_i\rangle\langle\tilde{r}_i|\otimes|\tilde{r}_i'\rangle\langle\tilde{r}_i'|,\ i = 1, 2 \quad (1)$$

with $r_i,\ \tilde{r}_i \in [0, 1]$, $r_i + \tilde{r}_i = 1$. This mixed state is a statistical mixture of two vectors $|r_i\rangle \otimes |r_i'\rangle$ and $|\tilde{r}_i\rangle \otimes |\tilde{r}_i'\rangle$ with the classical probability r_i and $\tilde{r}_i$, respectively. We show that the existing results in Refs. [11] also hold for these bipartite mixed states. The vectors fulfill the following relations,

$$\begin{aligned}\langle r_1|r_2\rangle &= s,\ \langle\tilde{r}_1|\tilde{r}_2\rangle = \tilde{s},\ \langle r_1'|r_2'\rangle = s',\ \langle\tilde{r}_1'|\tilde{r}_2'\rangle = \tilde{s}',\\ \langle r_i|\tilde{r}_i\rangle &= \langle r_i'|\tilde{r}_i'\rangle = 0, \end{aligned} \quad (2)$$

where $0 < s,\ s',\ \tilde{s},\ \tilde{s}' < 1$. We assume that the support space of the two states do not overlap, namely,

$$\langle r_1|\tilde{r}_2\rangle = \langle\tilde{r}_1|r_2\rangle = \langle r_1'|\tilde{r}_2'\rangle = \langle\tilde{r}_1'|r_2'\rangle = 0. \quad (3)$$

The unambiguous state discrimination of two general mixed states with arbitrary support space is hard to be handled and solved analytically [14]. Assumptions (2) and (3) ensure that the mixed states are not entangled and have no global coherence. In the next section, we consider the role of entanglement (global coherence) in discriminating pure states superposed via the vectors $|r_i\rangle \otimes |r_i'\rangle$ and $|\tilde{r}_i\rangle \otimes |\tilde{r}_i'\rangle$.

Here, in each round, one of the two bipartite states (1) is sent to two observers, Alice and Bob. The first (second) particle of the bipartite system, with local orthonormal basis $\{|r_i\rangle, |\tilde{r}_i\rangle\}$ $(\{|r_i'\rangle, |\tilde{r}_i'\rangle\})$, is provided for Alice (Bob). Alice performs a local operation on the first particle with POVM operators

$$\begin{aligned} M_1^A &= c_1|r_2^\perp\rangle\langle r_2^\perp| + \tilde{c}_1|\tilde{r}_2^\perp\rangle\langle\tilde{r}_2^\perp|,\\ M_2^A &= c_2|r_1^\perp\rangle\langle r_1^\perp| + \tilde{c}_2|\tilde{r}_1^\perp\rangle\langle\tilde{r}_1^\perp|,\\ M_0^A &= I^A - M_1^A - M_2^A, \end{aligned} \quad (4)$$

where $\{|r_1^\perp\rangle,\ |\tilde{r}_1^\perp\rangle\}$ and $\{|r_2^\perp\rangle,\ |\tilde{r}_2^\perp\rangle\}$ are bases orthogonal to $\{|r_1\rangle,\ |\tilde{r}_1\rangle\}$ and $\{|r_2\rangle,\ |\tilde{r}_2\rangle\}$, respectively. c_i and $\tilde{c}_i$ are non-negative real numbers less than 1.

Alice's POVM (4) must satisfy the following three properties: (a) $M_i^A \geq 0$, $(i = 0, 1, 2)$, (b) $M_1^A + M_2^A + M_0^A = I^A$, and (c) $\mathrm{Tr}[\rho_i(M_j^A \otimes I^B)] = 0$ for $i, j = 1, 2$ and $i \neq j$. The first two relations are the positive-semidefinite and completeness conditions of the POVM, respectively. The last one guarantees no error occurring in state discrimination. The operators M_1^A and M_2^A are for the conclusive outcomes (corresponding to the succeeding results) while M_0^A is for the inconclusive one (failure result).

From assumptions (2) and (3), one knows that the subspace spanned by $\{|r_1^\perp\rangle,\ |r_2^\perp\rangle\}$ is orthogonal to the one spanned by $\{|\tilde{r}_1^\perp\rangle,\ |\tilde{r}_2^\perp\rangle\}$. Thus, the POVM in Eq. (4) contains the direct sum "$\oplus$" and Eq. (4) can be written as

$$
\begin{aligned}
M_1^A &= c_1|r_2^\perp\rangle\langle r_2^\perp| \oplus \tilde{c}_1|\tilde{r}_2^\perp\rangle\langle\tilde{r}_2^\perp|,\\
M_2^A &= c_2|r_1^\perp\rangle\langle r_1^\perp| \oplus \tilde{c}_2|\tilde{r}_1^\perp\rangle\langle\tilde{r}_1^\perp|,\\
M_0^A &= M_{0s}^A \oplus \tilde{M}_{0s}^A = (I_s^A - c_1|r_1^\perp\rangle\langle r_1^\perp| - c_2|r_2^\perp\rangle\langle r_2^\perp|) \oplus (\tilde{I}_s^A - \tilde{c}_1|\tilde{r}_1^\perp\rangle\langle\tilde{r}_1^\perp| - \tilde{c}_2|\tilde{r}_2^\perp\rangle\langle\tilde{r}_2^\perp|).
\end{aligned}
\tag{5}
$$

Here, I_s^A and $\tilde{I}_s^A$ are the identity matrices on their respective subspaces. The POVM with the form of Eqs. (5) guarantees that the discrimination of mixed states can be carried out in two independent subspaces.

Set

$$q_i^A = \langle r_i|M_{0s}^A|r_i\rangle, \quad \tilde{q}_i^A = \langle\tilde{r}_i|\tilde{M}_{0s}^A|\tilde{r}_i\rangle.$$

We have

$$q_i^A = 1 - c_i(1-s^2), \quad \tilde{q}_i^A = 1 - \tilde{c}_i(1-\tilde{s}^2), \tag{6}$$

where $s^2 \le q_i^A \le 1, \tilde{s}^2 \le \tilde{q}_i^A \le 1$ $(i = 1, 2)$.

As a positive-semidefinite operator, the POVM operator M_0^A satisfies $\det M_0^A \ge 0$ [14], requiring that

$$q_1^A q_2^A - s^2 \ge 0, \quad \tilde{q}_1^A\tilde{q}_2^A - \tilde{s}^2 \ge 0. \tag{7}$$

If all information encoded in the first particle of the bipartite state is extracted by Alice, relations (7) become equalities. Otherwise, they are strict inequalities.

The Kraus operators K_i^A $(i = 0, 1, 2)$ corresponding to Alice's POVM ($M_i^A = K_i^{A\dagger}K_i^A$) are given by [14]

$$
\begin{aligned}
K_1^A &= \sqrt{c_1}|v_1\rangle\langle r_2^\perp| + \sqrt{\tilde{c}_1}|\tilde{v}_1\rangle\langle\tilde{r}_2^\perp|,\\
K_2^A &= \sqrt{c_2}|v_2\rangle\langle r_1^\perp| + \sqrt{\tilde{c}_2}|\tilde{v}_2\rangle\langle\tilde{r}_1^\perp|,\\
K_0^A &= (\sqrt{a_1}|v_1\rangle\langle r_2^\perp| + \sqrt{a_2}|v_2\rangle\langle r_1^\perp|) \oplus (\sqrt{\tilde{a}_1}|\tilde{v}_1\rangle\langle\tilde{r}_2^\perp| + \sqrt{\tilde{a}_2}|\tilde{v}_2\rangle\langle\tilde{r}_1^\perp|),
\end{aligned}
\tag{8}
$$

where $a_i = q_i^A/(1-s^2)$, $\tilde{a}_i = \tilde{q}_i^A/(1-\tilde{s}^2)$, $i = 1, 2$.

The postmeasured state σ_i $(i = 1, 2)$ corresponding to Alice's failure result can be expressed as

$$
\begin{aligned}
\sigma_i &= \frac{(K_0^A \otimes I^B)\rho_i(K_0^{A\dagger} \otimes I^B)}{\mathrm{Tr}[(K_0^A \otimes I^B)\rho_i(K_0^{A\dagger} \otimes I^B)]}\\
&= v_i|v_i\rangle\langle v_i| \otimes |r_i'\rangle\langle r_i'| + \tilde{v}_i|\tilde{v}_i\rangle\langle\tilde{v}_i| \otimes |\tilde{r}_i'\rangle\langle\tilde{r}_i'|.
\end{aligned}
\tag{9}
$$

Here, $\{|v_i\rangle, |\tilde{v}_i\rangle\}$ is the orthonormal basis of σ_i satisfying

$$
\begin{aligned}
&\langle v_1|v_2\rangle = t,\ \langle\tilde{v}_1|\tilde{v}_2\rangle = \tilde{t},\ \langle v_1|\tilde{v}_2\rangle = \langle\tilde{v}_1|v_2\rangle = 0,\\
&\langle v_1'|\tilde{v}_2'\rangle = \langle\tilde{v}_1'|v_2'\rangle = 0.
\end{aligned}
\tag{10}
$$

v_i and $\tilde{v}_i$ are the eigenvalues of σ_i,

$$v_i = \frac{q_i^A r_i}{q_i^A r_i + \tilde{q}_i^A\tilde{r}_i}, \quad \tilde{v}_i = \frac{\tilde{q}_i^A\tilde{r}_i}{q_i^A r_i + \tilde{q}_i^A\tilde{r}_i}. \tag{11}$$

The equivalence of the two expressions (5) and (8) ($M_0^A = K_0^{A\dagger}K_0^A$) requires

$$q_1^A q_2^A = s^2/t^2,\ \tilde{q}_1^A\tilde{q}_2^A = \tilde{s}^2/\tilde{t}^2. \tag{12}$$

Then, one has that q_i^A ($\tilde{q}_i^A$) is lower bounded by s^2/t^2 ($\tilde{s}^2/\tilde{t}^2$).

The success probability for Alice to identify her state is

$$
\begin{aligned}
P^A &= \sum_{i=1}^{2} P_i \mathrm{Tr}[\rho_i(M_i^A \otimes I^B)]\\
&= 1 - \sum_{i=1}^{2}(P_i r_i q_i^A + P_i\tilde{r}_i\tilde{q}_i^A).
\end{aligned}
\tag{13}
$$

According to Eqs. (7) and (12), the optimal discrimination occurs when $t = \tilde{t} = 1$. Hence, relation (12) turns out to be

$$q_1^A q_2^A = s^2,\ \tilde{q}_1^A\tilde{q}_2^A = \tilde{s}^2 \tag{14}$$

with the parameters q_i^A and $\tilde{q}_i^A$ satisfying

$$q_i^A \in [s^2, 1],\ \tilde{q}_i^A \in [\tilde{s}^2, 1]. \tag{15}$$

The failure probability of Alice is easily acquired as

$$P^{A(f)} = 1 - P^A = P_1 Q_1^A + P_2 Q_2^A, \tag{16}$$

where $Q_i^A = r_i q_i^A + \tilde{r}_i\tilde{q}_i^A$ $(i = 1, 2)$.

If Alice succeeds in discriminating her state, the procedure ends. Otherwise, another observer, Bob, performs unambiguous discrimination via optimal local POVMs on the second particle lying in the subspace spanned by the basis $\{|r_i'\rangle, |\tilde{r}_i'\rangle\}$. Then, the *a priori* probability of Bob's states is

$$P_{fi} = \frac{P_i Q_i^A}{P_1 Q_1^A + P_2 Q_2^A}, \quad i = 1, 2. \tag{17}$$

Construction of the optimal POVMs for Bob is similar to the ones for Alice in Eqs. (4)-(15) with the parameters c_i and $\tilde{c}_i$ replaced by c_i' and $\tilde{c}_i'$. q_i^A and $\tilde{q}_i^A$ are also replaced by q_i^B and $\tilde{q}_i^B$, respectively, where

$$q_i^B = 1 - c_i'(1 - s'^2), \quad \tilde{q}_i^B = 1 - \tilde{c}_i'(1 - \tilde{s}'^2).$$

We also have the constraints

$$q_1^B q_2^B = s'^2, \; \tilde{q}_1^B \tilde{q}_2^B = \tilde{s}'^2, \tag{18}$$

with $q_i^B \in [s'^2, 1]$ and $\tilde{q}_i^B \in [\tilde{s}'^2, 1]$.

The success probability for Bob to identify his state is

$$\begin{aligned} P^B &= \sum_{i=1}^{2} P_{fi} \mathrm{Tr}[\sigma_i (I^A \otimes M_i^B)] \\ &= \sum_{i=1}^{2} P_{fi} [c_i' v_i (1 - s'^2) + \tilde{c}_i' \tilde{v}_i (1 - \tilde{s}'^2)] \\ &= \sum_{i=1}^{2} P_{fi} [v_i (1 - q_i^B) + \tilde{v}_i (1 - \tilde{q}_i^B)]. \end{aligned} \tag{19}$$

Then, the failure probability for Bob's discrimination is given by

$$P^{B(f)} = 1 - P^B = P_{f1} Q_1^B + P_{f2} Q_2^B, \tag{20}$$

where $Q_i^B = v_i q_i^B + \tilde{v}_i \tilde{q}_i^B$ $(i = 1, 2)$. Thus, we obtain the total failure probability

$$P^{A(f)} P^{B(f)} = P_1 Q_1^A Q_1^B + P_2 Q_2^A Q_2^B. \tag{21}$$

According to relations (14), (18), and (21), the total successful probability of the LOCC protocol is

$$\begin{aligned} P_L &= 1 - P^{A(f)} P^{B(f)} \\ &= 1 - \sum_{i=1}^{2} (P_i r_i q_i^L + P_i \tilde{r}_i \tilde{q}_i^L), \end{aligned} \tag{22}$$

where $q_i^L = q_i^A q_i^B$, $\tilde{q}_i^L = \tilde{q}_i^A \tilde{q}_i^B$ $(i = 1, 2)$, and $q_1^L q_2^L = s_0^2$, $\tilde{q}_1^L \tilde{q}_2^L = \tilde{s}_0^2$ with $s_0 = ss'$ and $\tilde{s}_0 = \tilde{s}\tilde{s}'$. It can be easily found that the result of Eq. (21) is invariant under exchanging Alice and Bob.

The success probability for the global scheme is given by

$$P_G = 1 - \sum_{i=1}^{2} (P_i r_i q_i^G + P_i \tilde{r}_i \tilde{q}_i^G), \tag{23}$$

where the parameters q_i^G, $\tilde{q}_i^G$ $(i = 1, 2)$ satisfy the following relations:

$$q_1^G q_2^G = s_0^2, \; \tilde{q}_1^G \tilde{q}_2^G = \tilde{s}_0^2. \tag{24}$$

The result in Eq. (23) is of the same form as the one for Alice's local scheme in Eq. (13) with the parameters q_i^A, $\tilde{q}_i^A$, s, and $\tilde{s}$ replaced by q_i^G, $\tilde{q}_i^G$, s_0, and $\tilde{s}_0$, respectively.

Comparing the result of Eq. (22) with Eq. (23), it is obvious that the local scheme is equivalent to the global one. Setting $r_i = 1$, we derive the pure state $|\Psi_i\rangle = |r_i\rangle \otimes |r_i'\rangle$ $(i = 1, 2)$. The overlap $\langle\Psi_1|\Psi_2\rangle$ can be divided into two parties s and s'. The relation $\langle\Psi_1|\Psi_2\rangle = ss'$ implies that the difficulty for extracting information in local state discrimination (two-step procedure) is identical to that in the global scheme (one-step procedure). This condition guarantees the equivalence of these two schemes. Relations (22) and (23) indicate that the successful probability for discrimination of a mixed state ρ in Eq. (1) is equivalent to a weighted average of the one for two pairs of pure states $|r_1\rangle \otimes |r_1'\rangle$, $|r_2\rangle \otimes |r_2'\rangle$ and $|\tilde{r}_1\rangle \otimes |\tilde{r}_1'\rangle$, $|\tilde{r}_2\rangle \otimes |\tilde{r}_2'\rangle$, lying in their respective subspaces that are orthogonal to each other. That is a key reason why equivalence of local and global schemes also occurs for the mixed states.

The optimal success probability of global mixed-state discrimination, perfectly achieved by the local one, is shown in Table I for $P_1 r_1 \leq P_2 r_2$ and $P_1 \tilde{r}_1 \leq P_2 \tilde{r}_2$. The results are divided into four categories. For the both-states-identified case in the bottom right corner of Table I, we have $P^{(opt)} = 1 - 2\sqrt{P_1 P_2} F(\rho_1, \rho_2)$ where $F(\rho_1, \rho_2) = \sqrt{r_1 r_2} ss' + \sqrt{\tilde{r}_1 \tilde{r}_2} \tilde{s}\tilde{s}'$ is the fidelity [29] between ρ_1 and ρ_2. Here, the fidelity can be used as a generalized "inner product" to characterize the discrimination of mixed states. It can also be easily concluded that as one mixed state is neglected in this optimal solution (the case in the top left corner of Table I), the optimal successful probability is $P_2(1 - s_0^2)$ which is independent of r_i $(\tilde{r}_i)$ for $s_0 = \tilde{s}_0$.

From the above results one sees that conditions (2) and (3) greatly simplify the discrimination of mixed states (1). Nevertheless, the POVM for more general cases, in which the constraints in Eqs. (2) and (3) on Bob's state are relaxed, can still be constructed.

In more general cases, the vectors $|r_1'\rangle$, $|r_2'\rangle$, $|\tilde{r}_1'\rangle$, and $|\tilde{r}_2'\rangle$ overlap with each other. In order to discriminate the states unambiguously, Bob's POVM can be constructed in the form of Eq. (4),

$$\begin{aligned} M_i^{B*} &= c_i' |\alpha_i\rangle\langle\alpha_i| + \tilde{c}_i' |\tilde{\alpha}_i\rangle\langle\tilde{\alpha}_i|, \\ M_0^{B*} &= I - M_1^{B*} - M_2^{B*}, \end{aligned} \tag{25}$$

where c_i' and $\tilde{c}_i'$ $(i = 1, 2)$ are also non-negative parameters which satisfy $0 < c_i', \; \tilde{c}_i' < 1$. The vectors $|\alpha_i\rangle$ and

Overlap	$s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$	$s_0 \leq \sqrt{\frac{P_1 r_1}{P_2 r_2}}$
$\tilde{s}_0 > \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$	$P^{\max} = 1 - P_1 - P_2(r_2 s_0^2 + \tilde{r}_2 \tilde{s}_0^2)$	$P^{\max} = 1 - 2\sqrt{P_1 r_1 P_2 r_2} s_0 - P_1 \tilde{r}_1 - P_2 \tilde{r}_2 \tilde{s}_0^2$
$\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$	$P^{\max} = 1 - P_1 r_1 - P_2 r_2 s_0^2 - 2\sqrt{P_1 \tilde{r}_1 P_2 \tilde{r}_2} \tilde{s}_0$	$P^{\max} = 1 - 2\sqrt{P_1 r_1 P_2 r_2} s_0 - 2\sqrt{P_1 \tilde{r}_1 P_2 \tilde{r}_2} \tilde{s}_0$

TABLE I: Optimal success probability $P^{\max}$ of global mixed state discrimination in terms of P_i, r_i, $\tilde{r}_i$, s_0 and $\tilde{s}_0$. For $s_0 \leq \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$, we have $q_1^G = \sqrt{\frac{P_2 r_2}{P_1 r_1}} s_0$ ($\tilde{q}_1^G = \sqrt{\frac{P_2 \tilde{r}_2}{P_1 \tilde{r}_1}} \tilde{s}_0$) while $q_1^G = 1$ ($\tilde{q}_1^G = 1$) for $s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$ ($\tilde{s}_0 > \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$) corresponding to this optimal solution. In the former case, both of the mixed states are identified; while the latter case is considered to be one-state-identified because the success probability for identifying both $|r_1\rangle\langle r_1| \otimes |r_1'\rangle\langle r_1'|$ and $|\tilde{r}_1\rangle\langle\tilde{r}_1| \otimes |\tilde{r}_1'\rangle\langle\tilde{r}_1'|$ equals to zero [14, 18, 21]. If only one of $|r_1\rangle\langle r_1| \otimes |r_1'\rangle\langle r_1'|$ and $|\tilde{r}_1\rangle\langle\tilde{r}_1| \otimes |\tilde{r}_1'\rangle\langle\tilde{r}_1'|$ is neglected and the other one is identified, we say that the mixed state ρ_1 is partially identified (e.g. the case for $s_0 \leq \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 > \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$ with the optimal success probability achieved at $q_1^G = \sqrt{\frac{P_2 r_2}{P_1 r_1}} s_0$, $\tilde{q}_1^G = 1$).

$|\tilde{\alpha}_i\rangle$ can be acquired as [18],

$$
\begin{aligned}
|\alpha_i\rangle &= \frac{\sum_{j=1}^{2} G^{-1}_{2j-1,2i-1}|r_j'\rangle + G^{-1}_{2j,2i-1}|\tilde{r}_j'\rangle}{||\sum_{j=1}^{2} G^{-1}_{2j-1,2i-1}|r_j'\rangle + G^{-1}_{2j,2i-1}|\tilde{r}_j'\rangle||}, \\
|\tilde{\alpha}_i\rangle &= \frac{\sum_{j=1}^{2} G^{-1}_{2j-1,2i}|r_j'\rangle + G^{-1}_{2j,2i}|\tilde{r}_j'\rangle}{||\sum_{j=1}^{2} G^{-1}_{2j-1,2i}|r_j'\rangle + G^{-1}_{2j,2i}|\tilde{r}_j'\rangle||},
\end{aligned}
\tag{26}
$$

where G is the Gram matrix [14, 18] given by the four vectors $|r_i'\rangle$, $|\tilde{r}_i'\rangle$ ($i = 1, 2$):

$$
G = \begin{bmatrix}
\langle r_1'|r_1'\rangle & \langle r_1'|\tilde{r}_1'\rangle & \langle r_1'|r_2'\rangle & \langle r_1'|\tilde{r}_2'\rangle \\
\langle \tilde{r}_1'|r_1'\rangle & \langle \tilde{r}_1'|\tilde{r}_1'\rangle & \langle \tilde{r}_1'|r_2'\rangle & \langle \tilde{r}_1'|\tilde{r}_2'\rangle \\
\langle r_2'|r_1'\rangle & \langle r_2'|\tilde{r}_1'\rangle & \langle r_2'|r_2'\rangle & \langle r_2'|\tilde{r}_2'\rangle \\
\langle \tilde{r}_2'|r_1'\rangle & \langle \tilde{r}_2'|\tilde{r}_1'\rangle & \langle \tilde{r}_2'|r_2'\rangle & \langle \tilde{r}_2'|\tilde{r}_2'\rangle
\end{bmatrix}. \tag{27}
$$

It is easily verified that $\mathrm{Tr}[\sigma_i(I^A \otimes M_j^B)] = 0$ ($i,\ j = 1, 2, i \neq j$). Thus, Bob's success probability can also be acquired according to Eq. (19). Nevertheless, because the subspace spanned by $\{|\alpha_1\rangle, |\alpha_2\rangle\}$ is not orthogonal to the one spanned by $\{|\tilde{\alpha}_1\rangle, |\tilde{\alpha}_2\rangle\}$ anymore, it is difficult to optimize the success probability. We have the following conjecture.

[Conjecture 1]. For a fixed fidelity between ρ_1 and ρ_2, the success probability for discriminating the mixed states is impaired by the overlaps of the vectors $|r_i'\rangle$ and $|\tilde{r}_j'\rangle$ ($i, j = 1, 2$).

Namely, the local scheme is inferior to the global one. However, it is difficult to prove the general case for the conjecture, as Bob's success probability has a complex form. In Appendix A, we give a proof for a special case in which the overlaps are the same as in Eqs. (2) and (3), with only one of the zero overlaps replaced by $\langle r_2'|\tilde{r}_2'\rangle = \varepsilon$.

III. SIMULATION FOR MIXED STATE DISCRIMINATION

In the above results, the vectors are mixed via classical probabilities r_i and $\tilde{r}_i$ ($i = 1, 2$). In this part, we turn to study the case in which they are coherently superposed into the pure states. Namely, we are simulating mixed-state discrimination by identifying a pair of pure entangled (coherent) states of the form

$$
|\Psi_i\rangle = \sqrt{r_i}|r_i\rangle \otimes |r_i'\rangle + \sqrt{\tilde{r}_i}|\tilde{r}_i\rangle \otimes |\tilde{r}_i'\rangle, \quad i = 1, 2 \tag{28}
$$

occurring with the *a priori* probability P_i. The parameters r_i and $\tilde{r}_i$ are neither 1 nor zero. $|\Psi_i\rangle$ are both entangled and globally coherent, with the same fidelity as the mixed states (1) shown as

$$
F(|\Psi_1\rangle, |\Psi_2\rangle) = F(\rho_1, \rho_2) = \sqrt{r_1 r_2} s s' + \sqrt{\tilde{r}_1 \tilde{r}_2} \tilde{s} \tilde{s}'. \tag{29}
$$

For the local scheme, we adopt the same protocol as the one in Fig. 1, and the local POVMs M_i and Kraus operators K_i ($i = 0, 1, 2$) used by Alice and Bob given in Eqs. (4) and (8); we have Alice's failure probability

$$
\begin{aligned}
P_E^{A(f)} &= P_1\langle\Psi_1|M_0^A \otimes I^B|\Psi_1\rangle + P_2\langle\Psi_2|M_0^A \otimes I^B|\Psi_2\rangle \\
&= P_1 r_1 q_1^A + P_1 \tilde{r}_1 \tilde{q}_1^A + P_2 r_2 q_2^A + P_2 \tilde{r}_2 \tilde{q}_2^A.
\end{aligned} \tag{30}
$$

Corresponding to Alice's failure result, the postmeasured state for Bob is given by

$$
\frac{K_0 \otimes I|\Psi_i\rangle}{||K_0 \otimes I|\Psi_i\rangle||} = \sqrt{v_i}|v\rangle \otimes |r_i'\rangle + \sqrt{\tilde{v}_i}|\tilde{v}\rangle \otimes |\tilde{r}_i'\rangle, \tag{31}
$$

occurring with the *a priori* probability

$$
P_i^0 = \frac{P_i r_i q_i^A + P_i \tilde{r}_i \tilde{q}_i^A}{P_1 r_1 q_1^A + P_1 \tilde{r}_1 \tilde{q}_1^A + P_2 r_2 q_2^A + P_2 \tilde{r}_2 \tilde{q}_2^A} \quad i = 1, 2, \tag{32}
$$

where v_i and $\tilde{v}_i$ are given in Eq. (11).

Bob's failure probability

$$P_E^{B(f)} = P_1^0 v_1 q_1^B + P_1^0 \tilde{v}_1 \tilde{q}_1^B + P_2^0 v_2 q_2^B + P_2^0 \tilde{v}_2 \tilde{q}_2^B, \quad (33)$$

is the same as Eq. (20). The total failure probability $P_E^{A(f)} P_E^{B(f)}$ is identical to $P^{A(f)} P^{B(f)}$ in Eq. (21). Consequently, this simulation is perfect for the local scheme. But for the global scheme, we have a completely different conclusion.

[Theorem 1]. For the global scheme, the optimal success probability of discriminating the pure entangled states [Eq. (28)] is achieved perfectly by the result of mixed states for the both-states-identified case, $s_0 \leq \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$, and for the one-state-identified case, $s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 > \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$ under the condition $\sqrt{r_1 \tilde{r}_2}\tilde{s}_0 = \sqrt{r_2 \tilde{r}_1} s_0$. Otherwise, the results are superior to the ones of mixed states.

[Proof]. The global scheme is to discriminate a pair of nonorthogonal states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ with an inner product $s^* = \langle\Psi_1|\Psi_2\rangle = \sqrt{r_1 r_2} s_0 + \sqrt{\tilde{r}_1 \tilde{r}_2}\tilde{s}_0$. This comes down to an optimization problem:

$$\text{maximize } P_{SUCC} = 1 - P_1 q_1^0 - P_2 q_2^0, \quad (34)$$

$$\text{subject to } q_1^0 q_2^0 = s^{*2},\ q_1^0, q_2^0 \in [s^{*2},\ 1]. \quad (35)$$

We have the optimized success probability

$$\text{(i)}: P_{(E)G}^{\max} = 1 - 2\sqrt{P_1 P_2} s^* \quad \text{when } s^* \leq \sqrt{\frac{P_1}{P_2}}, \quad (36a)$$

$$\text{(ii)}: P_{(E)G}^{\max} = P_2(1 - s^{*2}) \quad \text{when } s^* > \sqrt{\frac{P_1}{P_2}}. \quad (36b)$$

Let us compare this result with the one of mixed-state discrimination in Table I. The four cases corresponding to different value ranges of s_0, $\tilde{s}_0$, and s^* are shown in Fig. 2.

Case (i) ($s_0 \leq \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$). We can easily obtain $s^* \leq \sqrt{P_1/P_2}$. Here, both the mixed and pure entangled states are all optimally identified. The optimized successful probability for discriminating mixed states is given by

$$\begin{aligned} P^{\max} &= 1 - 2\sqrt{P_1 r_1 P_2 r_2} s - 2\sqrt{P_1 \tilde{r}_1 P_2 \tilde{r}_2} \tilde{s} \\ &= 1 - 2\sqrt{P_1 P_2} s^* = P_{(E)G}^{\max}. \end{aligned} \quad (37)$$

Case (ii) ($s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 > \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$). We can also easily acquire $s^* > \sqrt{\frac{P_1}{P_2}}$. The optimal discrimination of both pure-entangled and mixed states is the one-state-identified case. According to the result in Eq. (36b) and

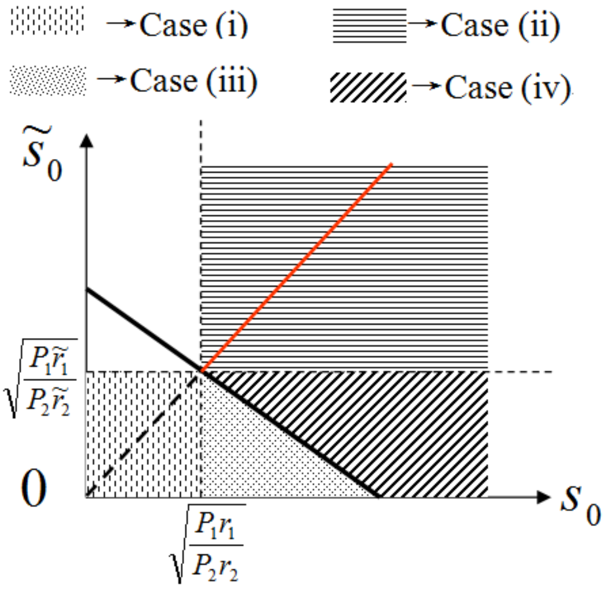

FIG. 2: Four regions corresponding to cases (i), (ii), (iii), and (iv), respectively, with different values of s_0, $\tilde{s}_0$, and s^*. The black solid line stands for $s^* = \sqrt{r_1 r_2} s_0 + \sqrt{\tilde{r}_1 \tilde{r}_2}\tilde{s}_0 = \sqrt{\frac{P_1}{P_2}}$ with fixed r_i and $\tilde{r}_i$ ($i = 1,\ 2$). The red solid line stands for $\sqrt{r_1 \tilde{r}_2}\tilde{s}_0 = \sqrt{r_2 \tilde{r}_1} s_0$.

the Cauchy-Schwarz inequality, one has

$$\begin{aligned} P_{(E)G}^{\max} &= P_2(1 - s^{*2}) \\ &= P_2[1 - (\sqrt{r_1 r_2} s_0 + \sqrt{\tilde{r}_1 \tilde{r}_2}\tilde{s}_0)^2] \\ &\geq P_2\{1 - (r_1 + \tilde{r}_1)(r_2 s_0^2 + \tilde{r}_2 \tilde{s}_0^2)\} \\ &= 1 - P_1 - P_2(r_2 s_0^2 + \tilde{r}_2 \tilde{s}_0^2) = P^{\max}. \end{aligned} \quad (38)$$

When $\sqrt{r_1 \tilde{r}_2}\tilde{s}_0 = \sqrt{r_2 \tilde{r}_1} s_0$ (red solid line in Fig. 2), the relation becomes an equality.

Case (iii) ($s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$, $s^* \leq \sqrt{\frac{P_1}{P_2}}$). The optimal discrimination of pure-entangled (mixed) states is the both-state-identified (one-state-partially-identified) case. We have

$$\begin{aligned} \Delta P &= P_{(E)G}^{\max} - P^{\max} \\ &= (1 - 2\sqrt{P_1 P_2} s^*) \\ &\quad -(1 - P_1 r_1 - P_2 r_2 s_0^2 - 2\sqrt{P_1 \tilde{r}_1 P_2 \tilde{r}_2}\tilde{s}_0) \\ &= (\sqrt{P_1 r_1} - \sqrt{P_2 r_2} s_0)^2 > 0. \end{aligned} \quad (39)$$

Case (iv) ($s_0 > \sqrt{\frac{P_1 r_1}{P_2 r_2}}$, $\tilde{s}_0 \leq \sqrt{\frac{P_1 \tilde{r}_1}{P_2 \tilde{r}_2}}$, $s^* > \sqrt{\frac{P_1}{P_2}}$). The optimal discrimination of pure-entangled (mixed) states is the one-state-identified (one-state-partially-identified) case. We have

$$\begin{aligned}\Delta P &= P_{(E)G}^{\max} - P^{\max} \\ &= P_2(1-s^{*2}) \\ &\quad -(1-P_1r_1-P_2r_2s_0^2-2\sqrt{P_1\tilde{r}_1P_2\tilde{r}_2}\tilde{s}_0) \\ &= F(\tilde{s}_0) = A\tilde{s}_0^2+B\tilde{s}_0+C, \end{aligned} \tag{40}$$

where

$$\begin{aligned}&A=-P_1(1-r_1)(1-r_2),\\ &B=2\sqrt{(1-r_1)(1-r_2)}(\sqrt{P_1P_2}-P_2-P_2\sqrt{r_1r_2}s_0),\\ &C=-(1-r_1)(P_1-P_2r_2s_0^2).\end{aligned} \tag{41}$$

The ΔP given in Eq. (40) can be viewed as a quadratic function of the variable $\tilde{s}_0$ with $\tilde{s}_0 \in (0, \sqrt{\frac{P_1\tilde{r}_1}{P_2\tilde{r}_2}}]$. Because of $A<0$, the minimum of ΔP is obtained at the boundary points, $\Delta P_{\min} = \min\{\Delta P|_{\tilde{s}_0\to 0},\ \Delta P|_{\tilde{s}_0=\sqrt{\frac{P_1\tilde{r}_1}{P_2\tilde{r}_2}}}\}$.

According to the constraint $s^* > \sqrt{\frac{P_1}{P_2}}$ with $\tilde{s}_0 \to 0$, we have

$$\sqrt{r_1r_2}s_0 > \sqrt{\frac{P_1}{P_2}}. \tag{42}$$

Then, we can easily get

$$\begin{aligned}\Delta P|_{\tilde{s}_0\to 0} &= -(1-r_1)(P_1-P_2r_2s_0^2)\\ &> -(1-r_1)(P_1-P_2r_2\frac{P_1}{r_1r_2P_2})\\ &= \frac{P_1(1-r_1)^2}{r_1} > 0.\end{aligned} \tag{43}$$

For another boundary point $\tilde{s}_0 = \sqrt{\frac{P_1\tilde{r}_1}{P_2\tilde{r}_2}}$, according to Eqs. (40) and (41), we have

$$\Delta P|_{\tilde{s}_0=\sqrt{\frac{P_1\tilde{r}_1}{P_2\tilde{r}_2}}} = (1-r_1)(\sqrt{P_2r_2}s_0-\sqrt{P_1r_1})^2 > 0. \tag{44}$$

Due to the symmetry of exchanging s_0 and $\tilde{s}_0$, another one-state-partially-identified case, $s_0 \le \sqrt{\frac{P_1r_1}{P_2r_2}}$, $\tilde{s}_0 > \sqrt{\frac{P_1\tilde{r}_1}{P_2\tilde{r}_2}}$, for the optimal discrimination of mixed states leads to the same conclusions as cases (iii) and (iv). □

Obviously, the difference between the optimal success probability for globally discriminating the pure and mixed states equals the one between the local and global schemes for the pure-entangled-state protocol itself. This difference arises from the coherent superposition of bipartite vectors. It also indicates that entanglement plays a key role in the process of global discrimination scheme for the pure states (28). Then, let us consider the difference $\Delta P = P_{(E)G}^{\max} - P^{\max}$ as a function of the entanglement $E(|\Psi_i\rangle)$ between the two particles. Set $r_1=r_2=r$. Based on the negativity entanglement measure [22], we have $E(|\Psi_i\rangle) = 2\sqrt{r(1-r)}$. From Fig. 3 one can see that the difference ΔP increases with the entanglement (as well as the global coherence), which is more obvious for the case (ii) (one-mixed-state-identified case). Consequently, for global pure state discrimination, entanglement between the two particles is a kind of critical recourse which is completely destroyed by local operations. Thus, the pure-entangled-state protocol cannot reflect any superiority versus mixed-state one in the local scheme. The effect of entanglement (global coherence) vanishes for special cases mentioned in *Theorem 1* where successful simulation occurs.

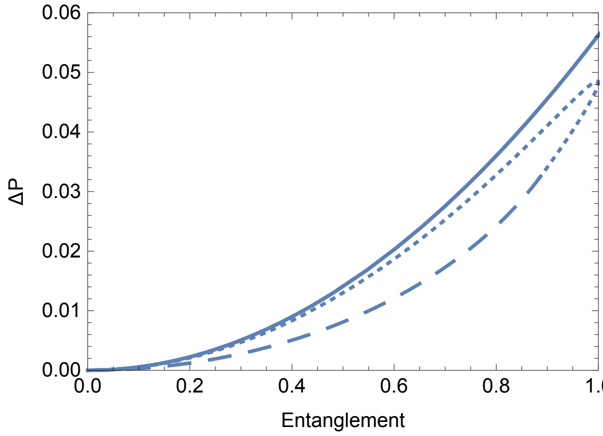

FIG. 3: The difference of the optimal success probability ΔP between the two schemes as functions of the entanglement $E(|\Psi_i\rangle)$ between the two particles corresponding to the cases for $P_1=0.1$, $P_2=0.9$, $s_0=0.7$, and $\tilde{s}_0=0.2$. Solid line, case (ii); dotted line, case (iii); dashed line, case (iv).

From the above results, it is indicated that relation (29) is a necessary condition for successful simulation. If this condition is violated, the results differs. For example, suppose that the pure entangled state in (28) is changed into

$$|\psi_i\rangle = \sqrt{r_i}|r_i\rangle\otimes|r_i'\rangle + \exp(i\phi_i)\sqrt{\tilde{r}_i}|\tilde{r}_i\rangle\otimes|\tilde{r}_i'\rangle,\ i=1,2, \tag{45}$$

with ϕ_i as a phase factor satisfying $\phi_2 \neq \phi_1 + 2k\pi$ for some integer k; for the global scheme we have

$$\begin{aligned}F(|\Psi_1\rangle,|\Psi_2\rangle) &= |\langle\psi_1|\psi_2\rangle|\\ &= \left|\sqrt{r_1r_2}s_0+\exp[i(\phi_2-\phi_1)]\sqrt{\tilde{r}_1\tilde{r}_2}\tilde{s}_0\right|\\ &< |\sqrt{r_1r_2}s_0| + \left|\sqrt{\tilde{r}_1\tilde{r}_2}\tilde{s}_0\right|\\ &= \sqrt{r_1r_2}s_0+\sqrt{\tilde{r}_1\tilde{r}_2}\tilde{s}_0\\ &= s^* = F(\rho_1,\rho_2).\end{aligned} \tag{46}$$

Here, the optimal success probability in discriminating the states is bound to be superior to the result in Eq. (36). That is, even for case (i), this simulation fails.

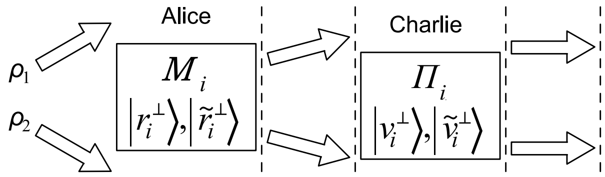

FIG. 4: SSD protocol presented in Refs. [14, 15]. First, a mixed quantum state ρ_i $(i = 1, 2)$ prepared with the *a priori* probability P_i is sent to Alice. Alice performs unambiguous discrimination on the state via nonoptimal global POVMs $\{M_i\}$ $(i = 0,\ 1,\ 2)$. Then, Alice sends her postmeasured state σ_i to Charlie and Charlie identifies σ_i with an optimal POVM on the same particle. The classical communications between Alice and Charlie are forbidden in this procedure.

IV. UNIFIED VIEW OF SSD AND LOCC PROTOCOL

The protocol of SSD mentioned in Ref. [6] has been extended to mixed initial states in [14]. We suppose that one prepares a mixed state (1) and sends it to Alice (see Fig. 4). Alice's POVMs and postmeasured states are of the same form as Eqs. (4) and (9). Here, Alice's POVMs are nonoptimal, meaning that $t \neq 1$ and $\tilde{t} \neq 1$. Namely, after Alice's discrimination, there is some information left in her state. Then, the postmeasured state is sent to another observer, Charlie. Charlie will discriminate the state via POVMs on the same particle, different from the LOCC protocol in Sec. II. The POVMs are given by

$$\begin{aligned}\Pi_1 &= \frac{1-q_1^C}{1-t^2}|v_2^\perp\rangle\langle v_2^\perp| + \frac{1-\tilde{q}_1^C}{1-\tilde{t}^2}|\tilde{v}_2^\perp\rangle\langle\tilde{v}_2^\perp|,\\ \Pi_2 &= \frac{1-q_2^C}{1-t^2}|v_1^\perp\rangle\langle v_1^\perp| + \frac{1-\tilde{q}_2^C}{1-\tilde{t}^2}|\tilde{v}_1^\perp\rangle\langle\tilde{v}_1^\perp|,\\ \Pi_0 &= I - \Pi_1 - \Pi_2, \end{aligned} \tag{47}$$

where $\{|v_1^\perp\rangle, |\tilde{v}_1^\perp\rangle\}$ and $\{|v_2^\perp\rangle, |\tilde{v}_2^\perp\rangle\}$ are bases orthogonal to $\{|v_1\rangle, |\tilde{v}_1\rangle\}$ and $\{|v_2\rangle, |\tilde{v}_2\rangle\}$, respectively. Here, $q_i^C = \langle v_i|\Pi_0|v_i\rangle$, $\tilde{q}_i^C = \langle\tilde{v}_i|\Pi_0|\tilde{v}_i\rangle$, $i = 1, 2$.

Charlie's discrimination is optimal, in the sense that $\det\Pi_0 = 0$, i.e., $q_1^C q_2^C - t^2 = 0$ and $\tilde{q}_1^C\tilde{q}_2^C - \tilde{t}^2 = 0$. The joint success probability for both Alice and Charlie to identify the states is

$$\begin{aligned}P_{SSD}^{A(s),C(s)} &= \sum_{i=1}^{2} P_i \mathrm{Tr}[\rho_i M_i]\mathrm{Tr}[\sigma_i\Pi_i]\\ &= \sum_{i=1}^{2} P_i[r_i(1-q_i^A)(1-q_i^C) + \tilde{r}_i(1-\tilde{q}_i^A)(1-\tilde{q}_i^C)].\end{aligned} \tag{48}$$

Its optimization of $P_{SSD}^{A(s),C(s)}$ has been given in Ref. [14].

During the procedure of SSD, classical communications are forbidden [14, 15, 20, 21]. This is essentially different from the local scheme where Bob's discrimination of the second particle is dependent on the premise that Alice communicates her failure result to him. The outcomes about Alice's succeeding, Bob's failing, or both succeeding are rejected by the LOCC scheme. Despite this distinction, we can show that the SSD and local protocol can be interpreted in a unified way: The information Alice and Charlie extract in the process of SSD is equivalent to that encoded in the first and second particle in LOCC which is distributed to Alice and Bob respectively. Then, we have the following theorem.

[Theorem 2]. If the POVMs used by the observer Bob (in local scheme) and Charlie (in the SSD scheme) satisfy $q_i^B = q_i^C$ and $\tilde{q}_i^B = \tilde{q}_i^C$ $(i = 1, 2)$, the probability that at least one of Alice and Charlie succeeds [15, 21] in SSD is equal to the total succeeding probability of the LOCC protocol.

[Proof]. For the SSD protocol, the probability that at least one of Alice and Charlie succeeds in detecting the state is given by

$$P_{SSD}^{A,C(1)} = P_{SSD}^{A(f),C(s)} + P_{SSD}^{A(s),C(f)} + P_{SSD}^{A(s),C(s)}, \tag{49}$$

which includes three parts $P_{SSD}^{A(f),C(s)}$, $P_{SSD}^{A(s),C(f)}$, and $P_{SSD}^{A(s),C(s)}$, standing for the probability that Alice fails (succeeds), Bob succeeds (fails), and both succeed, respectively. By straightforward calculations, we have

$$\begin{aligned}P_{SSD}^{A(f),C(s)} &= \sum_{i=1}^{2}[P_i r_i q_i^A(1-q_i^C) + P_i\tilde{r}_i\tilde{q}_i^A(1-\tilde{q}_i^C)],\\ P_{SSD}^{A(s),C(f)} &= \sum_{i=1}^{2}[P_i r_i(1-q_i^A)q_i^C + P_i\tilde{r}_i(1-\tilde{q}_i^A)\tilde{q}_i^C].\end{aligned} \tag{50}$$

Combining Eqs. (48)-(50), one has

$$\begin{aligned}P_{SSD}^{A,C(1)} &= \sum_{i=1}^{2}[P_i r_i(1-q_i^A q_i^C) + P_i\tilde{r}_i(1-\tilde{q}_i^A\tilde{q}_i^C)]\\ &= 1 - \sum_{i=1}^{2}(P_i r_i q_i^A q_i^C + P_i\tilde{r}_i\tilde{q}_i^A\tilde{q}_i^C),\end{aligned} \tag{51}$$

which is equal to P_L in Eq. (22) for $q_i^B = q_i^C$. □

V. HYBRIDIZATION BETWEEN LOCC AND OTHER SCENARIOS

In Sec. III, it is indicated that coherent superposition of the bipartite vectors leads to the difference between local and global schemes in state discrimination. In this section, we hybridize LOCC with three protocols (SSD, reproducing, and discrimination after broadcasting) in order to see whether different information tasks (e.g. sequential observation or classical communications between different observers) contribute to this gap. The latter two scenarios are introduced in Refs. [14, 15] to compare with SSD in order to see the effect of classical communications on state discrimination.

(1) Reproducing protocol: The observer Alice performs

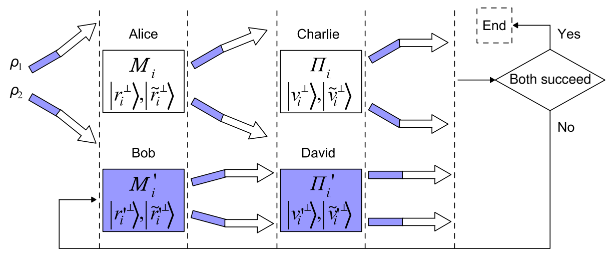

FIG. 5: Protocol for SSD via local operations. A bipartite quantum state ρ_i, $i = 1, 2$, prepared with the prior probability P_i, is sent to Alice. Alice and Charlie perform the SSD procedure on the first particle lying in the subspace spanned by the basis $\{|r_i\rangle, |\tilde{r}_i\rangle\}$ to discriminate the state. If both of them succeed, the procedure ends. Otherwise, the post-measured state is sent to other two observers, Bob and David, who perform another SSD procedure on the other particle to discriminate the state by their POVMs on the other subspace spanned by $\{|r_i'\rangle, |\tilde{r}_i'\rangle\}$.

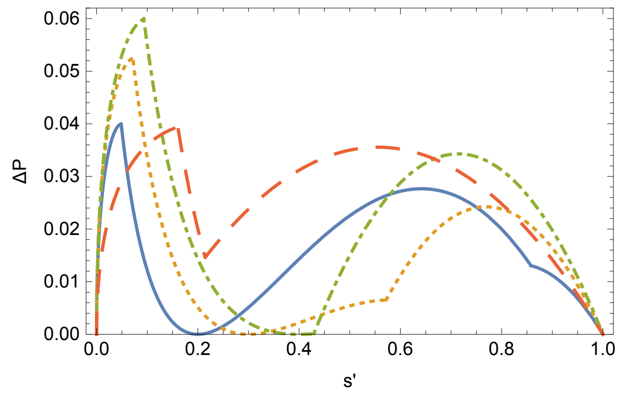

FIG. 6: The difference of the optimal success probability ΔP between global and local SSD vs s'. Solid line: $s = 0.2$; dotted line: $s = 0.3$; dot-dashed line: $s = 0.4$; dashed line: $s = 0.8$. If $s = s'$, the difference vanishes for a special value ranges of s and s' (e.g. $s = 0.2,\ 0.3,\ 0.4$), as shown in Appendix C. Otherwise, this difference is positive (e.g. $s = 0.8$).

an optimal unambiguous discrimination measurement on the quantum state ρ_i prepared with the probability P_i. If she succeeds, she reproduces the state and sends it to Charlie; if she fails, she informs Charlie that his measurement failed, and that is the end of the procedure.

(2) Discrimination after broadcasting: Broadcasting [14, 23] is identical to the quantum cloning [24] for the pure-state case. It transforms a mixed state ρ into ρ_{AC} satisfying $\mathrm{Tr}_A\rho_{AC} = \mathrm{Tr}_C\rho_{AC} = \rho$ with a certain success probability. If Alice succeeds in broadcasting, she shares the state ρ_{AC} with Charlie and they all perform optimal POVM on the partial states. If the broadcasting fails, she informs Charlie, and that is the end of the procedure.

The results in Refs. [14, 15] indicate that SSD performs better than these two strategies. In order to perform our hybridizations, four observers Alice, Bob, Charlie, and David will cooperate to discriminate the bipartite state (1) in three ways. The first (second) particle of the bipartite system is provided for Alice and Charlie (Bob and David).

(i) Hybridization of LOCC with SSD (see Fig. 5): Although classical communications are forbidden in the process of SSD, we suppose that Alice and Charlie are allowed to check their results with each other after they finish their measurements. If both of them succeed, the procedure ends. Otherwise, Bob and David perform another SSD procedure on the second particle of the bipartite system.

(ii) Hybridizing LOCC with protocols (1) and (2): Alice and Charlie perform the reproducing (discrimination after broadcasting) protocol on the first particle of our bipartite state ρ_i. If both of them succeed, the procedure ends. Otherwise, Bob and David perform another one on the second particle.

We enumerate examples for discriminating bipartite pure states (1) with $r_1 = r_2 = 1$. The difference of optimal successful probabilities ΔP between global and local SSD has been calculated in detail (see Appendixes B and C). For the hybridization of LOCC with reproducing protocol, the calculations of the optimal POVM is the same as the one in Sec. II. The *a priori* probability of the states left for Bob and David is equal to 1/2. Then, the difference of optimal success probability between the global and local schemes is given by

$$\begin{aligned} \Delta P^{(Re)} &= P^{opt}_{Re,G} - P^{opt}_{Re,L} \\ &= P^{opt(f)}_{Re,L} - P^{opt(f)}_{Re,G} \\ &= [1-(1-s)^2][1-(1-s')^2]-[1-(1-ss')^2] \\ &= 2ss'(1-s)(1-s') > 0. \end{aligned} \tag{52}$$

The successful probability for broadcasting two equal-prior pure states with the inner product s is $1/(1+s)$ [14, 15, 24]. Then, for the hybridization of LOCC with discrimination after broadcasting, the *a priori* probability of the state left for Bob and David is 1/2 as well. We also obtain the difference of optimal success probability between the global and local schemes:

$$\begin{aligned} \Delta P^{(Br)} &= P^{opt}_{Br,G} - P^{opt}_{Br,L} \\ &= [1-\frac{(1-s)^2}{1+s}][1-\frac{(1-s')^2}{1+s'}]-[1-\frac{(1-ss')^2}{1+ss'}] \\ &= \frac{2(1-s)(1-s')ss'(3+ss')}{(1+s)(1+s')(1+ss')} > 0. \end{aligned} \tag{53}$$

It is seen that using hybridization of LOCC with the other three protocols in which classical communication occurs to guarantee more observers to succeed extends the gap between the optimal success probability of the global and local schemes. We prove that for special cases, the result of global SSD can be achieved by the local one. In contrast, the local scheme is inferior to the global one for the other two protocols. Some of these results are given in Fig. 6.

VI. SUMMARY AND OUTLOOK

We have extended the local discrimination of bipartite pure states [11] to rank-2 mixed ones [14] via a statistical mixture of two pairs of state vectors. Assuming that these two vectors are orthogonal to each other and the support space of the two mixed states does not overlap, we have shown that the local scheme can perform as well as the scheme with global measurements, just as the result for pure initial states [11]. An example shows that the local scheme is inferior to the global one if this condition is not satisfied.

Then, the mixed (separable) state discrimination is simulated by pure entangled states, with the factors of classical probability in mixed states replaced by quantum probability amplitudes in pure states. It has been shown that this simulation is perfect for the local scheme because local POVM eliminates the entanglement and global coherence encoded in the pure entangled state. Thus, the pure-entangled-state protocol does not show any superiority to the mixed one. For the global scheme, successful simulation only occurs for a few special cases.

For the global scheme, this perfect simulation also occurs when the following two conditions are satisfied: (i) the fidelity of the mixed states equals that of the pure entangled states and (ii) both of the mixed states are identified. Otherwise, except for a few special one-state-identified cases, the mixed-state protocol is inferior to the pure-entangled-state one.

Concerning another SSD protocol given in Refs. [14, 15, 18] which is useful in quantum communication schemes (e.g., the B92 quantum cryptography protocol [5]), we have obtained an interesting result: the probability for at least one of the two observers succeeding in SSD is equal to the total succeeding probability of the local schemes. Thus, in spite of an essential distinction (classical communication is forbidden in SSD but required in the local scheme) between the two protocols, the SSD and LOCC protocols can be interpreted in a unified way.

At last, after hybridizing LOCC with the other three protocols (SSD, reproducing and broadcasting), we have found that the gap between the optimal success probabilities of the global and local schemes is extended. The result of the global scheme can be achieved by the local one only for SSD but not for the other two protocols.

We can easily get a generalized result for many-body systems. The successful probability is equivalent to the result of SSD in consecutive observers discussed partly in Ref. [25] if we require at least one observer succeeding. Namely, this unified view for discrimination in N-body states and SSD in N consecutive observers also holds. Simulation of N-body mixed states by pure entangled states can also be similarly studied. Moreover, by introducing an ancillary system coupled with the principal one [10, 20, 21, 26], the Hilbert space can be extended. Thus, a POVM can be realized via the tensor product method [27]. The role of quantum correlation [10, 20, 21] and coherence [26, 28] in pure-state discrimination has been also studied. It is also desirable to investigate the requirement of quantum correlations and coherence in mixed-state discrimination procedures via the Neumark formalism.

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APPENDIX A: A SPECIAL CASE FOR CONJECTURE 1

We consider a special case for the discrimination of mixed state (1) with a replaced condition $\langle r_2'|\tilde{r}_2'\rangle = \varepsilon$ that does not affect the fidelity between ρ_1 and ρ_2. Assume that $|r_1'\rangle = |0\rangle$ and $|\tilde{r}_1'\rangle = |1\rangle$. We have

$$\begin{aligned} |r_2'\rangle &= s'|0\rangle + \sqrt{1-s'^2}|2\rangle, \\ |\tilde{r}_2'\rangle &= \tilde{s}'|1\rangle + \frac{\varepsilon}{\sqrt{1-s'^2}}|2\rangle + \sqrt{1-\tilde{s}'^2-\frac{\varepsilon^2}{1-s'^2}}|3\rangle, \end{aligned} \quad \text{(A1)}$$

where ε satisfies $0 < \varepsilon \leq \sqrt{(1-s'^2)(1-\tilde{s}'^2)}$. According to Eq. (27), the Gram matrix can be written as

$$G = \begin{bmatrix} 1 & 0 & s' & 0 \\ 0 & 1 & 0 & \tilde{s}' \\ s' & 0 & 1 & \varepsilon \\ 0 & \tilde{s}' & \varepsilon & 1 \end{bmatrix}. \quad \text{(A2)}$$

Then, according to Eqs. (25), (26), (A2), and (19), Bob's success probability of discriminating the mixed states is given by

$$\begin{aligned} P^{B*} &= P_{f1}\mathrm{Tr}[\sigma_1(I^A \otimes M_1^{B*})] + P_{f2}\mathrm{Tr}[\sigma_2(I^A \otimes M_2^{B*})], \\ &= T(\varepsilon)\left(\frac{P_{f1}v_1c_1'}{1-\tilde{s}'^2-\varepsilon^2} + \frac{P_{f1}\tilde{v}_1\tilde{c}_1'}{1-s'^2-\varepsilon^2} + \frac{P_{f2}v_2c_2'}{1-\tilde{s}'^2} + \frac{P_{f2}\tilde{v}_2\tilde{c}_2'}{1-s'^2}\right), \end{aligned} \quad \text{(A3)}$$

where we have set $T(\varepsilon) = (1-s'^2)(1-\tilde{s}'^2) - \varepsilon^2$.

Compared with the result in Eq. (19), we obtain $P^{B*} < P^B$ and $\lim_{\varepsilon\to 0} P^{B*} = P^B$. The gap between P^{B*} and P^B still exists when the mixed states approach pure ones (e.g., v_1, $v_2 \to 1$). This fact shows the discontinuous points ($v_i = 0, 1$, $i = 1, 2$) of the success probability.

APPENDIX B: OPTIMAL SSD WITH BOTH STATES IDENTIFIED BY ALICE AND CHARLIE

For two initial bipartite pure states with $r_1 = r_2 = 1$, shared by Alice and Charlie and prepared with equal priority, we consider the optimal success probability of the local SSD protocol and compare it with the global one for the both-state-identified case of Alice and Charlie ($0 < s \leq 3 - 2\sqrt{2}$ [14, 20]).

Based on Eq. (48), for the local SSD protocol of pure states prepared with equal priority ($P_1 = P_2 = 1/2$), the optimization of the success probability $P^{A,C}_{SSD,L}$ for Alice and Charlie is given by the following:

$$\text{maximize } P^{A,C}_{SSD,L} = P_1(1-q_1^A)(1-q_1^C) + P_2(1-q_2^A)(1-q_2^C), \quad \text{(B1)}$$

$$\text{subject to } q_1^A q_2^C = \frac{s^2}{t^2},\ q_1^A, q_2^A \in [\frac{s^2}{t^2}, 1], \quad q_1^C q_2^C = t^2,\ q_1^C,\ q_2^C \in [t^2, 1]. \quad \text{(B2)}$$

The optimal success probability

$$P^{A,C(opt)}_{SSD} = (1-\sqrt{s})^2 \quad \text{(B3)}$$

occurs for $q_1^A = q_1^C = q_2^A = q_2^C = \sqrt{s}$, $t = \sqrt{s}$, for both-state-identified case ($0 < s \leq 3 - 2\sqrt{2}$) [14, 20]. The *a priori* probability of Bob's states is shown as

$$P_{fi} = \frac{P_i[1-(1-q_i^A)(1-q_i^C)]}{\sum_{i=1}^{2}\{P_i[1-(1-q_i^A)(1-q_i^C)]\}} = 1/2. \quad \text{(B4)}$$

In a similar way, the optimization of the success probability of Bob and David's local SSD can be given as follows:

$$\text{maximize } P^{B,D}_{SSD,L} = P_{f1}(1-q_1^B)(1-q_1^D) + P_{f2}(1-q_2^B)(1-q_2^D), \quad \text{(B5)}$$

$$\text{subject to } q_1^B q_2^B = \frac{s'^2}{t'^2},\ q_1^B, q_2^B \in [\frac{s'^2}{t'^2}, 1], \quad q_1^D q_2^D = t'^2,\ q_1^D,\ q_2^D \in [t'^2, 1]. \quad \text{(B6)}$$

Since the *a priori* probability of the two states for Bob and David is equal, the optimal successful probability of their SSD is [20]

$$\text{(i)}\quad P^{B,D(opt)}_{SSD,L} = (1-\sqrt{s'})^2 \quad \text{when } 0 < s' \leq 3 - 2\sqrt{2}, \quad \text{(B7a)}$$

$$\text{(ii)}\quad P^{B,D(opt)}_{SSD,L} = 1/2(1-s')^2 \quad \text{when } 3 - 2\sqrt{2} < s' < 1. \quad \text{(B7b)}$$

For case (i), the optimal SSD occurs at $q_1^B = q_1^D = \sqrt{s'}$, $t' = \sqrt{s'}$, whereas it occurs at $q_1^B = q_1^D = 1$, $t' = \sqrt{s'}$ for case (ii), where Bob and David conspire to ignore ρ_1. Then, the total failure probability corresponding to this optimal local SSD is

$$P^{opt(f)}_{SSD,L} = (1 - P^{A,C(opt)}_{SSD,L})(1 - P^{B,D(opt)}_{SSD,L}). \quad \text{(B8)}$$

The optimal success probability of the global scheme is equivalent to the result in Eqs. (B7) if we replace the inner product factors s' and $\tilde{s}'$ by ss' and $\tilde{s}\tilde{s}'$, respectively. The failure probability corresponding to the optimal global SSD is

$$\text{(i)}\quad P^{opt(f)}_{SSD,G} = 1 - (1-\sqrt{ss'})^2, \quad \text{when } 0 < ss' \leq 3 - 2\sqrt{2}; \quad \text{(B9a)}$$

$$\text{(ii)}\quad P^{opt(f)}_{SSD,G} = 1 - 1/2(1-ss')^2, \quad \text{when } 3 - 2\sqrt{2} < ss' < 1. \quad \text{(B9b)}$$

Nevertheless, here only case (i) in Eq. (B9a) is possible. Then, according to Eqs. (B3), (B7), (B8), and (B9a), the difference in the optimal success probabilities between global and local SSD ($\Delta P = P^{opt(f)}_{SSD,L} - P^{opt(f)}_{SSD,G}$) is given by

$$\text{(i)}\quad \Delta P = 2\sqrt{ss'}(1-\sqrt{s'})(1-\sqrt{s}), \quad \text{(B10a)}$$

$$\text{(ii)}\quad \Delta P = \sqrt{s}(1-\sqrt{s'})F(s,s'), \quad \text{(B10b)}$$

with $F(s,s') = (2-\sqrt{s})(1+\sqrt{s'})(1+s') - 4\sqrt{s'}$. Cases (i) and (ii) correspond to $0 < s' \leq 3-2\sqrt{2}$ and $3-2\sqrt{2} < s' < 1$, respectively. It can be easily acquired that ΔP is bound to be positive for case (i). Since $0 < s \leq 3 - 2\sqrt{2}$, for case (ii) we have $F(s,s') \geq F(3-2\sqrt{2}, s')$ and

$$\frac{dF(3-2\sqrt{2},s')}{ds'}|_{s'=s_0} = 0, \quad \frac{d^2F(3-2\sqrt{2},s')}{ds'^2}|_{s'=s_0} \approx 9.11 > 0 \quad \text{(B11)}$$

where $s_0 = (29 + 12\sqrt{2} - 2\sqrt{154 + 84\sqrt{2}})/63$. Hence, we get the minimum:

$$\min_{s'} F(3-2\sqrt{2}, s') = F(3-2\sqrt{2}, s_0) \approx 0.96 > 0. \quad \text{(B12)}$$

Therefore, we have $F(s,s') > 0$ and $\Delta P > 0$ according to relation (B10b).

APPENDIX C: OPTIMAL SSD WITH ONE-STATE-IDENTIFIED BY ALICE AND CHARLIE

For $3 - 2\sqrt{2} < s < 1$, the optimization of the result in Eq. (B1) is achieved as

$$P^{A,C(opt)}_{SSD} = 1/2(1-s)^2 \quad \text{(C1)}$$

for $q_1^A = q_1^C = 1$, $q_2^A = q_2^C = s$, $t = \sqrt{s}$, where ρ_1 is conspired to be ignored by Alice and Charlie [14, 20].

We obtain the *a priori* probabilities from the two states left for Bob and David:

$$P_{f1} = \frac{\frac{1}{2} - \frac{1}{2}(1-s)^2}{1 - \frac{1}{2}(1-s)^2}, \quad P_{f2} = \frac{\frac{1}{2}}{1 - \frac{1}{2}(1-s)^2}. \quad \text{(C2)}$$

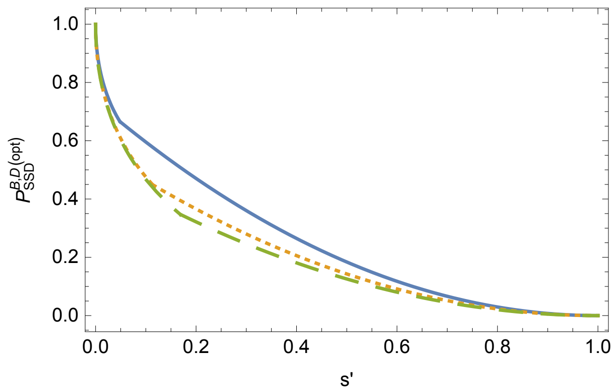

FIG. 7: The joint optimal success probability of Bob and David as a function of s'. Solid line: $s = 0.2$; dotted line: $s = 0.5$; dashed line: $s = 0.9$.

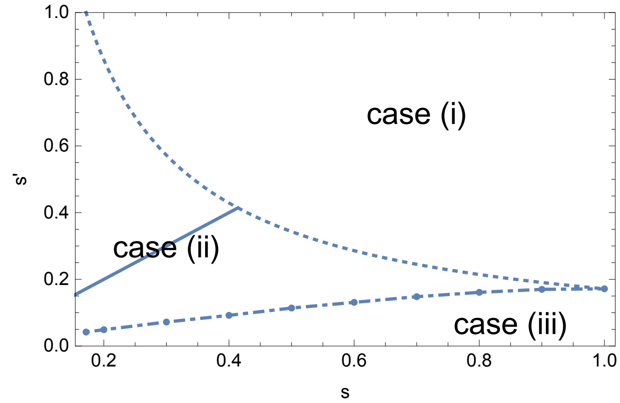

FIG. 8: The dotted line ($ss' = 3 - 2\sqrt{2}$) and dot-dashed line (a set for $s' = s^c$ plotted numerically corresponds to different values of s) are two bounds which give rise to three regions corresponding to cases (i) $s^c < s' < 1$, $3 - 2\sqrt{2} < ss' < 1$, (ii) $s^c < s' < 1$, $0 < ss' \leq 3 - 2\sqrt{2}$ and (iii) $0 < s' \leq s^c$, $0 < ss' \leq 3 - 2\sqrt{2}$, respectively. Only for case (ii) with $s = s'$ (solid line), the optimal successful probability of local SSD attains the result of global one.

Using a random search method [21], we can seek out the optimized success probability of SSD for both Bob and David. For fixed s, we have $P_{f1} \leq 1/2$. The optimized success probability occurs at $t' = \sqrt{s'}$ and $q_1^B = q_1^D$, which indicates the equivalence between the information extracted by Bob and David. The result of this optimization is given by

$$\begin{aligned}
&\text{(i) } P_{SSD}^{B,D(opt)} = P_{f1}(1-q^*)^2 + P_{f2}(1-\frac{s'}{q^*})^2, \\
&\quad \text{when } 0 < s' \leq s^c; \\
&\text{(ii) } P_{SSD}^{B,D(opt)} = P_{f2}(1-s')^2, \text{ when } s^c < s' < 1,
\end{aligned} \tag{C3}$$

where q^* satisfies $P_{f1}(q^*)^4 - P_{f1}(q^*)^3 + P_{f2}s'q^* - P_{f2}s'^2 = 0$, and the critical value s^c is determined by $P_{f1}(1-q^*)^2 + P_{f2}(1 - s^c/q^*)^2 = P_{f2}(1-s^c)^2$. For case (i), the optimal success probability occurs at $q_1^B = q_1^D = q^*$, while it occurs at $q_1^B = q_1^D = 1$ for case (ii), where Bob and David conspire to ignore the state ρ_1. In Fig. 7, it is shown that as s decreases, Bob's state tends to be equal prior. And the critical value s^c approaches its maximum $3 - 2\sqrt{2}$, which is consistent with the result in Ref. [21].

According to Eqs.(C1), (C3), and (B8), the total failure probability of the optimal local SSD protocol can also be obtained. For the global protocol, the failure probability of the optimal SSD can be obtained from the result in Eqs. (B9) with two possible outcomes. The difference of the optimal successful probability between the global and local protocols is acquired corresponding to the following three cases (i), (ii), and (iii), as shown intuitively in Fig. 8.

Case (i): $s^c < s' < 1$, $3 - 2\sqrt{2} < ss' < 1$. We have

$$\begin{aligned}
\Delta P &= [1-\frac{1}{2}(1-s)^2][1-P_{f2}(1-s')^2] - [1-\frac{1}{2}(1-ss')^2] \\
&= \frac{1}{2}(1-s)(1-s')(s+s'+ss'-1).
\end{aligned} \tag{C4}$$

Since $s' > \frac{3-2\sqrt{2}}{s}$, we get

$$\begin{aligned}
s' - \frac{1-s}{1+s} &> \frac{3-2\sqrt{2}}{s} - \frac{1-s}{1+s} \\
&= \frac{(s-\sqrt{2}+1)^2}{s(1+s)} \geq 0,
\end{aligned} \tag{C5}$$

from which we have $s + s' + ss' - 1 > 0$. Hence, from Eq. (C4), it is easily obtained that $\Delta P > 0$ as well.

Case (ii): $s^c < s' < 1$, $0 < ss' \leq 3 - 2\sqrt{2}$. We have

$$\begin{aligned}
\Delta P &= [1-\frac{1}{2}(1-s)^2][1-P_{f2}(1-s')^2] - [1-(1-\sqrt{ss'})^2] \\
&= \frac{1}{2}(\sqrt{s}-\sqrt{s'})^2[2-(\sqrt{s}+\sqrt{s'})^2] \geq 0.
\end{aligned} \tag{C6}$$

As $s = s'$, we get $\Delta P = 0$. Namely, the optimal success probability of the global SSD is attained by the local one.

Case (iii): $0 < s' \leq s^c$, $0 < ss' \leq 3 - 2\sqrt{2}$. This is a complicated case and is difficult to solve analytically. By numerical experiment via 10^5 random numbers, it can be ensured that ΔP is also larger than zero.

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