

# Error Estimation and Uncertainty Quantification for non-linear CFD problems

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Modern research and engineering rely on numerical simulations to predict the behaviour of fluids and some derived physical quantities of interest. These predictions are often strewn with errors and uncertainties. Numerical errors come from replacing the real physics with approximate models solved by numerical approximations, while uncertainties are due to insufficient knowledge of some input variables (i.e. design parameters or data), boundary/initial conditions or models parameters. Realistic CFD problems are complex systems often turbulent and/or shock-dominated. Proper error estimation for these problems is not a trivial task and remains a challenge for scientists due to strong non-linearity and ad-hoc turbulence models when subject to perturbations.

Our team effort is focused on both deterministic and stochastic error control. We will first present an introduction to a goal-based adaptive method aimed to reduce deterministic discretisation errors committed on a target of interest, with application to unsteady CFD problems governed by Euler and Navier-Stokes equations [1].

Several methods for improving the non-intrusive stochastic analysis for these complex problems are discussed in the context of uncertainty quantification. An adaptive stochastic spectral method is presented for problems with steep solution gradients developing in the parametric space [2]. This method is based on the expansion of the stochastic response in a piecewise generalised Polynomial Chaos (gPC) basis. Accurate approximation are obtained using sensitivity-based adaptivity. The convergence and accuracy of standard non-intrusive stochastic spectral method is an issue we have addressed via the formulation of a new iterative method particularly well suited when nonlinear transformations of random variables are in play and can be viewed as a new way of tracking Gibbs phenomenon.

The quantification of error committed in the prediction of different physical quantities in a more realistic turbulent problem is also investigated from a probabilistic point of view. We have analysed the propagation of errors in a LES setup for different quantities of interest the sensitivity to different simulation parameters (grid stretching, Smagorinski model constant, ...) through a gPC stochastic approach [4]. This analysis has been applied for an evolving flow, from a laminar regime highly dependent on the

inlet conditions to a fully developed turbulent regime. This approach allows a probabilistic characterisation of the error, as well as a quantification of its sensitivity to each considered parameter and to their combinations.

The problem of uncertainty propagation through a complex CFD problem where deterministic discretisation errors can be very important is one of our perspectives for the very near future. Indeed, we aim for a coupling of both issues, probably in a goal-based manner.

## References

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