Geometric Structures in Tensor Representations

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(joint work with W. Hackbusch and A. Nouy)

In this talk we introduce a tensor subspace based format for the representation of a tensor in a topological tensor space. To do this we use a property of minimal subspaces which allow us to describe the tensor representation by means of a rooted tree. By using the tree structure and the dimensions of the associated minimal subspaces, we introduce the set of tensors in a tree based format with either bounded or fixed tree based rank. This class contains the Tucker format and the Hierarchical Tucker format (including the Tensor Train format). In particular, any tensor of the topological tensor space under consideration admits best approximations in the set of tensors in the tree based format with bounded tree based rank. Moreover, we show that the set of tensors in the tree based format with fixed tree based rank is an analytical Banach manifold. This local chart representation of the manifold is often crucial for an algorithmic treatment of high-dimensional time-dependent PDEs and minimisation problems. However, in our framework, the tangent (Banach) space at a given tensor is not a closed complemented subspace in the natural ambient tensor Banach space. Therefore, we study the differential of the natural inclusion map as a morphism between Banach manifolds. It allows us to discuss the Dirac-Frenkel variational principle in the framework of topological tensor spaces.